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Journal
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Vol. 249 FEBRUARY, 1950 No. 2

# COMMUNICATIONS AND ELECTRONICS—THE MOST ESSENTIAL TOOLS OF JOINT COMMAND [1]

BY

EARL E. STONE [2]

I feel specially honored by this opportunity to address members of The Franklin Institute, the Institute of the Aeronautical Sciences, and the Philadelphia Chapter of the Armed Forces Communications Association.

For the last half century, the military services have been directly associated with the development of radio transmission and reception—the *radio art*—which has now grown into the *electronics art*. Because of the nature of some of my duty assignments, I have been personally able to observe many of the remarkable communications and electronics developments which have occurred during my active Naval service. Before getting down to my principal remarks, let me take you back briefly to an incident in 1917 when I was an Ensign just out of Annapolis. I think those of you who remember what radio materiel was like in World War I will appreciate the humor in this incident. I can chuckle when I think about it today, but in 1917, believe me, it was not a laughing matter to me—trying to learn about radio, and, of course, trying to please my commanding officer.

During the First World War, I served in a small cruiser—the first *U.S.S. Cleveland*. I was certainly busy on that assignment, being in charge of a group of 5-in. guns, and also regularly standing officer of the deck watches day and night, and in addition being the ship's communications officer. My captain in those days was an able but stern old skipper who demanded a very high standard of duty performance. I learned a great deal from him—the hard way—and frequently I had to do some explaining when things went wrong, as they sometimes did

[1] This is an address presented at a Joint Meeting with The Franklin Institute, of the Philadelphia Sections of the Institute of Aeronautical Sciences and of the Armed Forces Communications Association, held on Wednesday, November 2, 1949, at The Franklin Institute.

[2] Rear Admiral, U. S. Navy, Washington, D. C.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

even back in 1917. I recall one morning when we were convoying merchant ships across the Atlantic, my senior radio operator—who, incidentally, was a champion fleet boxer, but not a champion radioman—informed me that he had been unable to copy the radio press transmission during the night, because of too much static. Of course, I had to report that failure to the skipper and I knew how irate he would be to miss his news at breakfast.

Said the captain to me: "Well, young man, why did you fail to copy the Annapolis radio press broadcast last night?"

"Well, sir, the operator reported that there was too much static—he couldn't copy the press through the severe static interference."

"Too much interference from static did you say, Stone? Why, don't you know what kind of a radio receiver we have on this ship?"

"Yes, sir, the name plate on the receiver reads type number *one* P seven-six."

"Stone, that receiver, I'll have you know, is not marked one P seven-six but rather I (eye) P 76. Do you understand—that is an I and not a one, on the name plate. Furthermore, that IP means *interference preventer*. So don't ever tell *me* that you have static interference on *this* ship!"

You may be sure I never mentioned *static* to *that* captain again—but later I found out that he didn't like my mentioning our trouble with the *potentiometer* any better.

But to be serious—wonderful progress certainly has been made since I was first exposed to radio materiel in 1917.

Tonight I want to talk of communications and electronics primarily—but in order to make the importance of these military command tools clear, I feel it is desirable to discuss *briefly* several different but related matters, including such items as the importance of personnel, and the importance of research and development; the importance of the communication operating companies to the military services; what constitutes joint command, and the readiness of the three military services to meet a national emergency.

First then, a few words about personnel. I know that one of our common problems in the three services today is to maintain adequate trained personnel to utilize effectively, and to maintain, the communications-electronics material that has already been provided. Military aircraft, surface ships, submarines and tanks, as well as many military establishments on shore throughout the world, such as communication stations, radar stations and direction-finder stations have important, and often very complex communications-electronics installations. Without trained personnel that equipment would be of no use—in fact, it could even be a hindrance.

Our emergency sources of trained personnel are of three main categories: there is the all-important Communications-Electronics Industry, including the communication operating companies; there are the many

amateurs of the American Radio Relay League and the Military Amateur Radio System upon whom we know we can depend for their resourcefulness and unique "know how"; and there is our Electronics Reserve. I have the greatest faith in the men and women of our Communications-Electronics Industry, in our radio amateurs, and in the officers and enlisted personnel of our Electronics Reserve. I sometimes wonder, however, if all of these people realize how *very* much we will have to depend upon them to wage and win any future war, since the regular service men and women can be but a small part of the total personnel needed to meet wartime requirements? The regular nucleus is extremely important, of course, but it must be expanded many times, and quickly, when the emergency comes. Let me use a Navy example from World War II to illustrate. The prewar Naval Communications personnel was expanded about twenty times to meet World War II requirements. For any future war, we not only must have great numbers of trained persons ready for active service, but ready almost overnight. Accordingly, I believe that the best interests of the country will be served by having a large number of communication-electronics men in the Reserve Units of the Armed Forces. Their prior preparation and identification of interest would aid in the proper placing of these personnel in time of emergency. It would help our defense planning and also aid industry's preparation for such adjustments in their organization as would be required.

Our people, when at home, when at work, and when at play, are living in an electronic world—radio and television broadcast receivers, police radio systems, radio and electronics equipped commercial airplanes and trains, taxis, buses, and ships, and medical applications of electronics (including diathermy, X-ray, fluoroscope, cardiograph, and other medical aids), sound movies, hearing aids, photoradio and facsimile, burglar alarms, and electronic cookers—to name only some of the better known electronic devices. Future electronics applications seem to be limited only by the limit of the imagination of electronics engineers and scientists. Many people today have at least some electronic training and "know how," and that is a valuable national asset. Much must necessarily be expected from our trained technical people, such as many of you who are here tonight, who, by reason of your abilities, experience and technical training, will be able to contribute much more to national defense in the communications-electronics field (whether in the military services or in industry) than most men and women who may quickly don uniforms when war comes.

I wish to say a few words about *Joint Command*. Before World War II, Joint Command was a term used in war plans—it certainly was not a commonly used term; it rarely became a reality except in joint exercises. The war demonstrated the necessity for Joint Command as the *usual*, rather than the *unusual*, situation. There are now Joint Commands—and in this case, I mean three service unified

commands—throughout the world. The Army, Navy, and Air Force are under an Army General in the Caribbean; an Air Force General commands in Alaska; an Admiral commands in the Pacific; an Army General commands in the Far East; an Admiral commands in the Atlantic; and an Army General commands in Europe.

To be effective, Command, whether Joint or Single-Service, requires many supporting activities. There must, of course, be the best possible weapons. Assuming that we possess the necessary high quality weapons, including bombs, missiles, torpedoes, guns, tanks, submarines, and especially airplanes and surface ships of all required types, there are certain other activities upon which the exercise of Command is dependent, either directly or indirectly—for example, supply of ammunition, food, and fuel. There must also be medical activity to insure proper sanitation and good health, and there must be hospitals for the sick and wounded. Many other important supporting activities might be listed without which Command could not be exercised effectively.

Since the fighting forces must be readily controlled and directed, there *must* be effective communications. In fact, without adequate communications no commander would willingly risk action against an able adversary. If a question exists in anyone's mind as to the absolute necessity for communications as well as other electronics to utilize air power and sea power, he may be reminded of the fact that big bombers would not leave their bases and big carriers would not put to sea without assurance that their respective complex communications-electronics installations were fully ready for use. Incidentally, a current motion picture production, entitled, "Task Force," gives a very good idea of the dependence which Navy Command must place on communications and electronics facilities in wartime carrier operations.

Whereas logistic supply can usually be effected *before* the fighting starts, and thereafter usually not cause the Commander great concern during battle, communications and electronics are tools in the Commander's hands *during battle* which, if they should *not* prove effective, would cause the Commander the gravest concern. Some of you may possibly be wondering just what I have in mind when I mention communications and electronics as tools of command. At the risk of oversimplifying, I'll try to make that matter clear in a few words.

As for communications as a command tool, besides the skilled personnel involved, I am thinking about the channels over which information and orders may be passed rapidly between the High Commander and his Subordinate Commanders, and the channels connecting his fighting units—whether they be tanks, planes, ships or combat teams. Such communication channels form military wire and radio networks which are sometimes described as the nerve system for controlling the actions of the complex military body. A point to remember about military communications *is* that as the tempo and ferocity of modern warfare increases, what with faster airplanes, faster ships, atom bombs,

and guided missiles, at many times there is need for practically *instantaneous* communications rather than merely rapid communications.

When I speak of electronics as a command tool, besides the skilled personnel involved, I have in mind *materiel*—radio transmitters and receivers, radio direction-finders, various electronic equipment including fire-control gear, countermeasures jamming apparatus, radar for search and radar for navigation, and other electronic materiel which will assist the Commander in wartime.

Certain aspects of electronics are of *great* importance both to the Military and to Industry, although specific applications are naturally different. One such aspect is that of control devices, such as for pilotless aircraft and guided missiles; then there are telemetering applications, that is, methods of relaying data from an operating point to a practical control or observation point—for example, the transmission of data from a rocket or missile in flight to its control station. Then there are various mathematical computing devices, so-called "electronic brains" which can solve mathematical and other problems of very great complexity in a matter of seconds, or at the most in minutes, which heretofore took days, weeks and even months to solve.

Probably the most important *defense* application of electronics is in connection with air defense. Timely electronic warning of an approaching enemy air group—whether the warning be received on shore, in ships or in airplanes—might, if promptly disseminated and acted upon, save thousands of lives, and possibly even the nation.

In the joint communications-electronics field, unification is working very well indeed. There are common operating procedures in use in the three services, common communication instructions, common procurement procedures, a procedure for standardization of components of electronic equipment, and coordination of research and development. There are always bound to be some honest differences of opinion in such a complex field as military communications and electronics. But time and again such differences have been resolved satisfactorily among the Chiefs of Communications of the three services, and no doubt other problems will be so resolved in the future—by agreeing on what will be best for our national security.

Our national security depends upon scientific knowledge and the rapid conversion of scientific knowledge into new or improved weapons and techniques. If World War III should come, and of course I hope it never does, it will be fought with certain vastly improved weapons and techniques, rather than with those we knew in World War II. In this connection you will appreciate how vitally important it is for the United States to stress electronics research and development. I believe we in the United States have no superiors in the electronics world. But we must strive to stay out front in the electronics race, both in research and development, and in mass production, to reap the dividends in wartime that electronics superiority will surely give us. We must

realize that today *all* nations are aware of the importance of electronics in warfare. We dare not rest upon our oars at this stage of the research and development race.

Fortunately, there exists today in the High Commands of all three Services, an adequate appreciation of the great importance of communications and electronics. As most of you probably appreciate, electronics is the very heart of many of our most promising military projects. The airplane of World War I was equipped with its meager electronics gear just before it was ready to be flown. Not so today, when the electronics gear is assembled and shaped up, and then the plane is sometimes almost literally built around the electronics equipment. Without its load of electronics equipment the modern military plane couldn't hope to perform its mission. Recently I learned that one of the latest big bombers carries nearly a ton of electronics gear, all carefully designed and installed to meet its combat requirements. The job of providing and installing electronics equipment for big airplanes is obviously difficult. Installations in submarines present somewhat similar problems to those of big airplanes.

Now I wish to say a few words about the importance of the U.S. communication operating companies to national security. The cooperation of these U.S. companies during World War II was indeed splendid, and I know that there is no question about the support which the operating companies as well as the remainder of the communications-electronics industry will continue to give the Military Services. It is imperative in time of war for our nation to have an established effective and comprehensive worldwide commercial communications system to serve all of our country, but particularly to serve the Military Services, and to supplement military communications facilities. While the three Military Services together have the nucleus of a worldwide communication system in peacetime, they cannot possibly maintain a worldwide system adequate for war because of lack of men and money.

And now a few remarks about the recent appointment of a Director of Communications-Electronics by the Joint Chiefs of Staff, with the approval of the Secretary of Defense. The first Director is Major General Harold M. McClelland of the Air Force, whom many of you may know, and who is well qualified for this important coordination assignment. The directorship will rotate among the three Services, with a change every two years. The Director serves as Chairman of the Joint Communications-Electronics Committee, which has an enviable record of achievement. The aim of the Joint Communications-Electronics organization is to insure the fullest *coordination* among the three Services in the communications-electronics field—to effect economy without impairing combat or administrative efficiency.

While striving for the utmost *coordination* in our military communications, it should be appreciated that the *consolidation* of communi-

cations cannot be carried to extremes since each Military Service has specific essential needs for combat communications that cannot be fused into a single unified communications service. For example, in the Army and Marine Corps, the combat communications of ground troops; in the Navy, shore-to-ship, ship-to-ship, ship-to-plane operational communications; and in the Air Force, ground-to-air and plane-to-plane operational communications. An example where the utmost coordination of joint communications is required is in the complex communications necessary for a large joint amphibious landing.

It must be remembered that a primary purpose of unification is to improve operating efficiency and insure economy. However, individual service requirements will still be given full consideration in order when practicable to permit each service to exploit to the fullest any important advantage accruing to it by its nature or relative superiority. In determining what the composition of our unified military machine must be, in order to meet the most likely threats to our national security, it is necessary to relate weapons, forces, strategy and funds, so that the resulting three-Service fighting strength will be the greatest team strength that can be provided and maintained for the dollars which can be spent. It should be clear to all who have read the newspapers recently that no Service in peacetime is ever likely to get as many dollars as it feels it must have. We all realize that there must be a limit on the National Defense Funds. I can assure you, economy and efficiency are being stressed.

At the outbreak of any future war which is suddenly forced upon us, it seems likely that we will have only minimum three-Service Forces available. It appears that there will be more need than heretofore for joint, that is, three-Service operations under unified Commanders, because of the specialization or concentration of each Service on its currently assigned primary roles and missions. Each Service certainly will help the other two Services, to the maximum extent necessary and possible. Joint operations in the future will require optimum teamwork among the Services, and joint communications and electronics will surely play a most important role.

Let me remind you in concluding my remarks that the age-old fundamental principles of warfare do not change, that is to say, principles concerning *mobility*, *flexibility*, *versatility*, *surprise*, *precision*, and concentration of force. These have been found *sound* through the years, as weapons and techniques have changed *radically*. Today *mobility is the key to fighting power*, and *air power* is certainly the dominant element both in land and sea operations. But the basic objectives of war *do not change*, and thus always finally, the destruction of the enemy's capacity and his will to fight *must be achieved*. That is a job which requires an adequate Army, Navy and Air Force, operating effectively together as our National Defense Team.

**New Tension Meter Twice as Sensitive as Old One.**—A new and improved Tension Meter has been recently brought out by the Saxl Instrument Company of Harvard, Massachusetts. In addition to instant trigger action and correct calibration under performance conditions, the new model is equipped with ball bearings throughout. This makes roller friction negligible. Special precision pulleys recessed in the trigger mechanism automatically guide the yarn into position so that the meter can be inserted blind and no yarn can get caught behind the rollers. No "wiggling-in" is necessary. Because yarn contraction is negligible, gentle insertion and release are assured.

Improvements consist of introducing a transparent flywheel through which the readings are taken. This important feature makes steady readings possible even on quickly traversing machinery such as quill winders. Sustained accuracy is obtained because the permanently calibrated spring is always at rest when not in use. The meter is dynamically calibrated under actual performance conditions. This is far more practical than static testing.

The new model has twice the sensitivity of the old 250-gram model, which is still being manufactured. The Saxl Tensiometer has a total capacity of 125 grams. Fifty grams are read from the black figures on the first revolution of the dial. This gives gram sensitivity for sensitive nylons and other fine yarn. The remaining 50 to 125 grams are read from the red figures on the second revolution. Therefore, without special adjustments the pointer can overtravel and continue to give precise readings even when the tension is above the 50-gram range.

Double the sensitivity, extended capacity, quick trigger insertion (even in limited spaces) and freedom from oscillations make this meter ideally suitable for quick tension measurements on all yarns including nylon and rayon and on machines such as warpers, winders, slashers and for winding elastic threads, checking shuttle tension, etc.

The Saxl Tensiometer is also used for quick tension measurements on Radio Wire, etc. used for galvanometer coils, ammeter coils, and the handling of hair-thin wires (copper, tungsten, etc.) which are used in radio tubes, incandescent lamps, etc. Full information is available from the Saxl Instrument Company, Harvard, Massachusetts.

**A "flying laboratory" to test early-warning radar** equipment and procedures has been set up by the Navy aboard a modified Lockheed Constellation, in a move to extend the range of shipboard detection devices, limited by the horizon. Antennae for the plane's powerful radars are housed in two large radome humps midway along the top and bottom of the fuselage. Extra technicians will be aboard.

**Propulsion of aircraft by nuclear energy** was actively studied during 1949, and progress made in solving problems of materials, reactors, shielding, power cycles and airframe design configuration, according to Maj. Gen. Donald L. Putt, Air Force Director of Research and Development. The project is being continued jointly with the Atomic Energy Commission, the National Advisory Committee for Aeronautics, and the Navy.

# ORIGIN AND MEANING OF CIRCULAR AND HYPERBOLIC FUNCTIONS IN ELECTRICAL ENGINEERING

BY

A. BOYAJIAN [1]

SUMMARY

Sinusoidal currents in time, arising from impressed sinusoidal voltages in time, need no explanation; but circular and hyperbolic functions of unobvious circular and hyperbolic space angles arising apparently spontaneously in electrical circuits call for some explanation. Accordingly, the concept of an angle, adequate both for circular and hyperbolic functions, is first developed geometrically, pointing out that an angle is nothing but the percentage value (more precisely, percentage/100 or *per-unit* value) of an arc recognized as the *vector* increment of the radius vector. Naturally, the radius vector itself is used as the per-unit base, hence the name *radians* (meaning, radii) for this kind of measure numbers. The idea naturally leads to the compound-interest like calculation of the per-unit value of a gross change in the radius vector.

The vector concept recognizes as angles those incremental arcs that are parallel to the radius vector just as well as those that are normal. The former are called *hyperbolic* angles, the latter *circular*. Thus an arc that represents a circular angle and one that represents a hyperbolic angle are $j$ with respect to each other. Obviously, in the first case, only the direction of the radius vector changes; in the latter, only its magnitude.

Applied to a physical variable, this concept of an angle means that a change in the variable expressed in *per-unit* [2] is the angular value of that change in radians.

This concept of angles leads to logarithms and exponentials, yielding a simple interpretation of the logarithms of negative, imaginary and complex numbers so that, in its light, by inspection $\log(-1)$ is recognized as $\pi j$, $\log j$ is recognized as $(\pi/2)j$, etc.

The cosine of an angle $\theta$ (whether $\theta$ is circular, or hyperbolic, or mixed) is interpreted as the resultant of two variables, initially alike, but one having undergone the vector per-unit change $(+\theta)$, and the other $(-\theta)$. A sine is interpreted as the resultant of two variables, initially equal and opposite, and one having undergone the vector per unit change $(+\theta)$, the other $(-\theta)$. The origin of such pairing of vectors is found in reflection. In transmission lines, discontinuities like open circuits and short circuits act as perfect reflectors, and provide reflected variables which match the original variable in the indicated manner to yield with the original a resultant corresponding to the sine or cosine (or sinh or cosh) of the per-unit change involved.

## INTRODUCTION

In alternating current circuits, sinusoidal voltage is impressed and a sinusoidal current is obtained; and naturally enough no one feels that an explanation is needed. However, when a continuous (d-c.) voltage is impressed on a system and a sinusoidal or hyperbolic function of time or space (or both) is obtained, an explanation is called for. When the

[1] Apparatus Department, General Electric Company, Pittsfield, Mass.

[2] All such reckonings in this paper are to be understood as compound interest calculation.

transmission engineer talks about hyperbolic sines and cosines of $\alpha x$, $\alpha x$ representing the "hyperbolic angle" of the line at the point $x$, any listener is entitled to ask just what he visualizes by a hyperbolic angle and how a straight length of wire turns out to be such a thing, and how hyperbolic sines and cosines develop in it. To be satisfactory, the answer to such a question must be primarily physical. Any purely mathematical answer, without at least a geometrical visualization, would be evasion of the engineer's inquiry. Some authors have drawn on hyperbolas so as to develop a system of hyperbolic functions in analogy to trigonometric functions associated with circles, but such theories fail to explain where either the circle or the hyperbola is in a transmission line or in a circuit of $R$, $L$, $G$ and $C$.

The general problem may conveniently be considered in two steps. First, the origin and meaning of angles, and then the origin and meaning of their functions. The second step is not by any means obvious when the first is ostensibly made clear. To illustrate this, one could

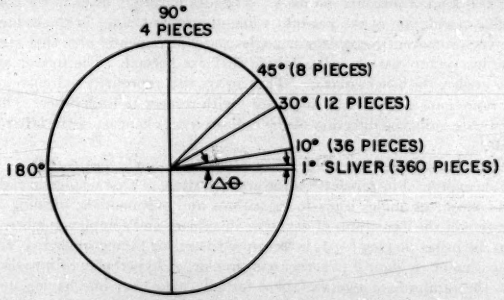


FIG. 1. Childhood concept of angles.

explain the circular angle intelligibly even to a child without providing a clue to circular functions. Thus, an uncut pie is 100 per cent. Some foolish people have called this hundred per cent 360°, because, with this measure, the pie becomes divisible in many ways without involving fractional numbers. If cut into two pieces, each piece is 180° (Fig. 1); if cut into three, 120°; four, 90°; five and seven pieces are not represented by whole numbers; but six, eight, nine and ten, even twelve and twenty and a whole lot more divisions up to 360 pieces are whole number of degrees. The last one—1°—is such a thin sliver that one could not taste it, though one could shave with it if it were steel. Anything smaller than that surely deserves to be considered a fractional thing. The sharpness or bluntness of the pieces of our pie, considered as a wedge, gives an idea of the angle they represent, but how such a notion of angles implies trigonometric functions is altogether unobvious.

### A BETTER THEORY OF ANGLES

At the college level, we define an angle by the length of the arc associated with it, and in order to be free to use an arc of any radius, we measure the angle not by the absolute length of the arc but by its *relative* length, with its radius as the unit of length. Thus, if an arc is given as so many inches, there is no hint of the size of the angle associated with it, but if it is given in terms of its radius, as so many "radiuses," or as so long "per unit radius", or, in the more conventionalized form of the word, as so many "*radians*," now the angle is defined, except for some important details as follows.

If we are talking about common (circular) angles, the little arc shown in Fig. 2 will be a measure of the little angle $\Delta\theta$ provided that, (*a*) the arc is perpendicular to the radius vector at every point; (*b*) the length of the arc is expressed "in terms of"[3] the radius vector, that is,

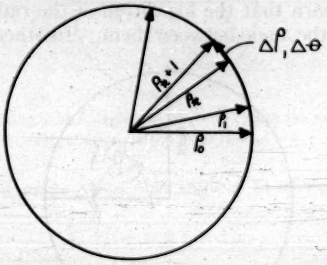


FIG. 2. Concept of an angle in terms of its arc, an element of the arc being treated as a *vector change in the radius vector*, as the latter changes.

as so many *radians* (or equivalent); and (*c*) the arc or angle must be understood as representing an increment or change in the radius vector itself. The last point means that the little arc must be considered as $\Delta\rho$, as otherwise all meaning would be lost. Accordingly, our equation for the little angle of Fig. 2 will be

$$\Delta\theta = \frac{\Delta\rho}{\rho}. \tag{1}$$

The fact that the arc must be perpendicular to the radius vector introduces the thought of a *vector* into our consideration; and it is easy to see from Fig. 2 that $\Delta\rho$ represents the *vector* change in the radius vector as it changes from $\rho_k$ to $\rho_{k+1}$. If $\Delta\rho$ is taken in sufficiently small lengths, the $\rho$ in the denominator of Eq. 1 will be the $\rho_k$ of Fig. 2.

Although usually we specify $\Delta\theta$ by its numerical value, yet Fig. 2

[3] Expressing one thing "in terms of" another means that this other will be used as the unit of measurement. I am surprised that this meaning of the words is not universally recognized as their only rigorous meaning.

shows that perpendicularity to the radius vector is implicit in its concept. We shall find it very wise and helpful to make this fact explicit. If in Eq. 1 we express $\Delta\rho$ by its true vector value, and treat $\rho$ as the unit reference vector, then $\Delta\theta$ will turn out to be a pure imaginary number, $jb$. Of course, we interpret $b$ as a vector parallel to the reference vector (the radius vector), and $j$ as representing 90° rotation (counter clockwise) *with respect to the radius vector*.

If we wish to measure the total angle between two positions, say $\rho_1$ and $\rho_2$, of the radius vector (Fig. 2), Eq. 1 leads to

$$\theta_{1-2} = \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho} = \log_e \rho_2 - \log_e \rho_1, \tag{2a}$$

$$= \log_e \frac{\rho_2}{\rho_1}. \tag{2b}$$

From (2$b$) we learn that the logarithm of the ratio of two values of the radius vector is the angle between them. In other words, logarithms

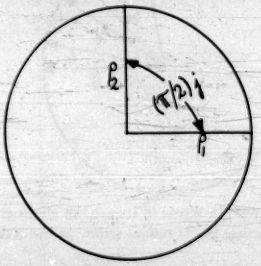


FIG. 3. The arc of a ninety-degree angle is $\pi/2$ times the radius vector in magnitude and perpendicular to it at every point. Therefore the complete specification of such an angle is $(\pi/2)j$. This then is the logarithm of $\rho_2/\rho_1$, hence the logarithm of $j$.

are another way of defining an angle. The first way was to define the angle by the arc; this second one is by the initial and final values of the radius vector. It follows that if we have the answer one way, we have the answer to the other way too. Let us consider some interesting practical consequences of this.

If the arc is the quadrant of a circle (Fig. 3), the "circular angle is $\pi/2$," or the "angle is $(\pi/2)j$." The equivalent phrases set off with quotation marks emphasize the thought that, to be explicit about the nature of $\theta$, we must use either the adjective "circular" or the symbol "$j$" with it. In conversation, the first may be more convenient; but in equations, the latter is the only logical choice. So the angle shown in Fig. 3 is $(\pi/2)j$. Considering the vector ratio of $\rho_2$ to $\rho_1$, it will be seen to be just $j$. Substituting these values of $\theta$ and $\rho_2/\rho_1$ into Eq. 2$b$,

we get:

$$\frac{\pi}{2}j = \log_e j. \tag{3}$$

Glory be! We have discovered the log of $j$, which is considered meaningless by many people.

Equation 2*b* says, more generally, that the log of the ratio of two numbers is the angle between them. Could this new insight then tell us also the log of $(-1)$ which we "*know*" (?) does not exist, because negative numbers don't have logs. Of course it does. As seen in Fig. 4,

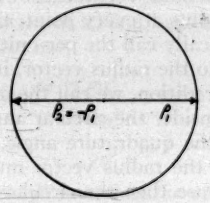


FIG. 4. In tracing a 180° angle, the angle is $\pi j$ and the ratio $\rho_2/\rho_1$ is $-1$. Therefore, $\pi j$ is the logarithm of $-1$.

$-1$ is the value of $\rho_2/\rho_1$ when the angle $\theta$ is $\pi j$ radians, so according to Eq. 2*b*

$$\pi j = \log (-1). \tag{4}$$

As simple as that.

As one further example, how about 45°? (Fig. 5).

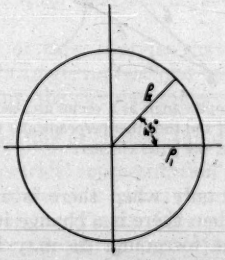


FIG. 5. For a 45° rotation, the angle is $(\pi/4)j$, and $\rho_2/\rho_1$ is $(1+j)/\sqrt{2}$. Therefore, $(\pi/4)j$ is the logarithm of $(1+j)/\sqrt{2}$.

Well, its angle is $(\pi/4)j$, and $\rho_2/\rho_1$ equals $(1+j)/\sqrt{2}$. Substituting these two values into Eq. 2*b*

$$(\pi/4)j = \log \{(1+j)/\sqrt{2}\}. \tag{5}$$

### ANOTHER KIND OF ANGLE

Having taken a vector view of things in the foregoing, we may well consider now what would happen if, in Eq. 1, $\Delta\rho$ were parallel to $\rho$ instead of being normal to it. Of course that would not define a circular angle any more, but from our broadened point of view it might represent another kind of angle, and it turns out to be the so-called hyperbolic angle.

Now that helps a lot toward simplifying things. Evidently, when an increment in the radius vector is expressed in per-unit values, with the radius as the unit of length, we are expressing the *angular* value of the change as so many radians, regardless of its direction. If the arc is *perpendicular* to the radius at every point, the radians are a quadrature number, and we logically call the per-unit change a circular angle. If the change is *parallel* to the radius vector, its radians value is a real number, and, following tradition, we call the angle a hyperbolic one.

One might wish to consider the circular angle as the real angle and the hyperbolic angle as the quadrature angle, but there is no getting away from the fact that the radius vector must be considered as the reference axis, in which case then the circular angle is the quadrature one.

When $\Delta\rho$ is parallel to $\rho$, and the angle is hyperbolic, it follows that $\rho$ is changing in magnitude, though not in phase. That is why circular

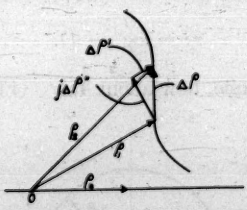


FIG. 6. Illustrating the complex angle as a vector arc, each element of the arc resolvable into two components, one parallel and the other perpendicular to the radius vector, the former representing the hyperbolic angle, the latter circular.

angles may be involved only when there is a change in phase, and hyperbolic angles only when there is a change in magnitude.

As a corollary to the foregoing, $\Delta\rho$ may be a complex number, expressible as $\Delta\rho' + j\Delta\rho''$; and $\rho_2$ and $\rho_1$ need not be of the same length (Fig. 6). Equations 1 and 2 still hold in a generalized sense, yielding a *complex* angle $\theta' + j\theta''$, $\theta'$ representing the hyperbolic component of the total angle, and $j\theta''$ its circular component. Thus,

$$\theta = \theta' + j\theta'' = \int_1^2 \frac{\Delta\rho}{\rho} = \int \frac{\Delta\rho' + j\Delta\rho''}{\rho}. \qquad (6)$$

Note that in this symbolism the figures for the components, $\theta'$, $\theta''$, $\Delta\rho'$, $\Delta\rho''$ are real numbers; while $\Delta\rho$ is a complex number; and $\rho$ *is always a real number* because, at each new point, the corresponding $\rho$ is the new reference vector for that point as the axis of reals, so that $\Delta\rho$ *must be resolved into its components with respect to* $\rho$. In this analysis, not the $x$-axis but the radius vector is the axis of reals. It doesn't matter that the axis of reals does not stay put, but changes from point to point. In compound interest, the principle does not stay put, but increases from year to year. The radians defined by Eq. 1 and 2 represent vectorially *compound* percentage change in the radius vector.

In order to extract the maximum information out of Eq. 2, let us make a simple transformation of it, and look at it again.

Solved for $\rho_2$, Eq. 2 becomes

$$\rho_2 = \rho_1\epsilon^{\theta} \tag{7a}$$

$$= \rho_1\epsilon^{\theta'+j\theta''} \tag{7b}$$

$$= \rho_1\epsilon^{\theta'}\epsilon^{j\theta''}. \tag{7c}$$

We learn here the strange fact that *the exponent of an exponential always represents an angle in radians!* No wonder then that an exponential represents an expanded or shrunk vector if the exponent is a real angle (that is, hyperbolic), and a rotating vector if the exponent is a quadrature angle (that is, circular).

In all cases, *the exponent represents vectorially the compound per-unit change in the radius vector in changing from its initial vector value to its final value.*

As a general conclusion to all the foregoing:

*The per-unit value of a vector change in a quantity is the angular value of the change in radians.*

The real component of this, that is, that parallel to the vector quantity, is the hyperbolic angle represented by the vector change; and the quadrature component, the circular angle.

## GEOMETRICAL INTERPRETATION OF CIRCULAR AND HYPERBOLIC FUNCTIONS

The well-known geometrical representation that the cosine is represented by the projection of the unit radius vector on the $X$-axis, and the sine by the projection on the $Y$-axis, fails in the case of hyperbolic angles. A clue to a more adequate representation is obtained by considering the exponential definitions of sines and cosines, whether circular or hyperbolic. Thus,

$$\cosh\theta = \frac{\epsilon^{\theta} + \epsilon^{-\theta}}{2}; \tag{8a}$$

$$\cos\theta = \frac{\epsilon^{j\theta} + \epsilon^{-j\theta}}{2}; \tag{8b}$$

$$\sinh \theta = \frac{\epsilon^{\theta} - \epsilon^{-\theta}}{2}; \tag{9a}$$

$$\sin \theta = \frac{\epsilon^{j\theta} - \epsilon^{-j\theta}}{2j}. \tag{9b}$$

Comparing (8*a*) with (8*b*), and (9*a*) with (9*b*), cos $\theta$ may also be considered cosh $j\theta$, and sin $\theta$ may also be considered $-j \sinh j\theta$.

Evidently, the only mathematical difference between the hyperbolic and circular quantities, whether angle or function, is just $j$; and that while either set can be expressed in terms of the other, it may well be

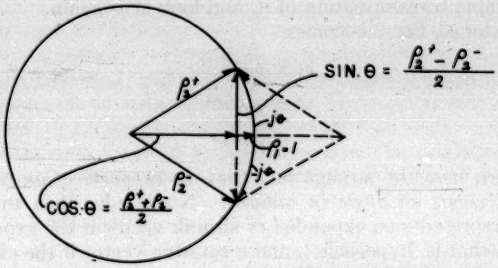


FIG. 7. Circular sines and cosines interpreted vectorially as the mean value of the radius vector for the positive and the negative values of the same circular angle: half the sum of the two radius vectors for cosine, half the difference for sine.

recognized that the hyperbolic way of looking and tagging is more logical. We should have had only one kind of geometry, and that should have used the hyperbolic nomenclature and symbols with a $j$ introduced where called for. This will be still clearer when we consider the geometry of these functions.

To simplify exposition, let the initial value $\rho_1$ of the radius vector in Eq. 7*a* be taken as 1 in. along the $X$-axis (Fig. 7). Then, for a positive angle $j\theta$, $\rho_2{}^+$ will equal vectorially $\epsilon^{j\theta}$, and for a negative angle of the

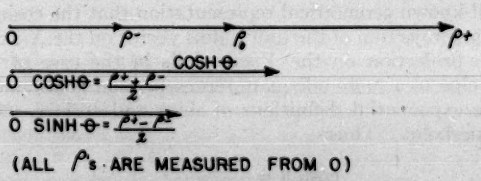


FIG. 8. Hyperbolic sines and cosines interpreted vectorially as the mean value of the radius vector for the positive and the negative values of the same hyperbolic angle: half the sum of the two radius vectors for the hyperbolic cosine, and half the difference for the hyperbolic sine.

same magnitude, $\rho_2^-$ will equal $\epsilon^{-j\theta}$. If we happen to be interested in the mean value of the two unit vectors, after one has undergone a per-unit change $+j\theta$, and the other $-j\theta$, then obviously what we are interested in will be, by Eq. 8, $\cosh j\theta$ or $\cos\theta$, as shown vectorially in Fig. 7. If one unit vector is negative with respect to the other, or for some reason must be subtracted from the other, then the mean resultant value we are looking for will be, by Eq. 9, $\sin\theta$ or $j\sinh j\theta$. This also is shown vectorially in Fig. 7, in which *the vector representing the sine will be seen to be normal to the vector representing the cosine*, strictly in accordance with the hyperbolic notation which is the only logical consistent notation.

Figure 8 illustrates the case when the change is parallel to the radius vector. It is interesting to observe that now both sine and cosine are real numbers, that is, vectors parallel to the radius vector.

Figure 9 illustrates the case when the angle is complex, and again

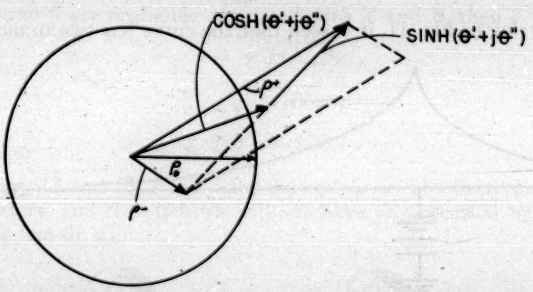


FIG. 9. Vectorial construction of the hyperbolic sine and cosine of a complex angle.

it may be seen that the final results we are interested in are the vector resultants of two unit vectors, one undergoing a specified positive per unit change, the other an equal negative per unit change. Their mean vector sum gives the hyperbolic cosine vectorially, and their mean difference the hyperbolic sine vectorially.

In the foregoing geometrical interpretation of circular and hyperbolic functions, we created a wholely artificial curiosity about the matter, justifiable only as a preparation for practical applications, and now it is time to do that.

### ORIGIN OF ANGLES AND THEIR FUNCTIONS IN ELECTRICAL ENGINEERING

*Example I. The Leaky Telegraph Line*

The distribution of current and voltage in a leaky telegraph line of indefinite length, with series resistance $R$ and shunt conductance $G$

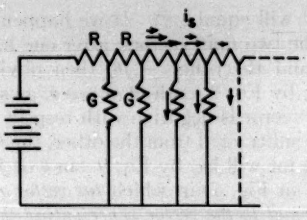


FIG. 10. A leaky telegraph line.

per unit length (Fig. 10), is known to be of the form

$$e = E_0\epsilon^{\pm\alpha x} \tag{10}$$

$$i = I_0\epsilon^{\pm\alpha x} \tag{11}$$

$$\alpha = \sqrt{RG}. \tag{12a}$$

Referring to Fig. 11, it is seen that the curve has two branches and

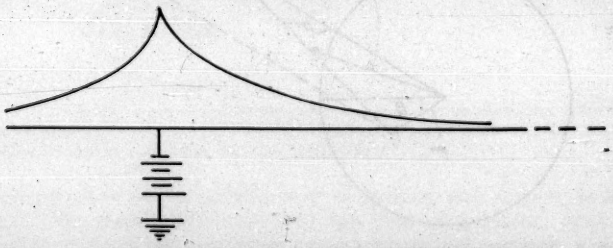

FIG. 11. Distribution of current and voltage in a line like that of Fig. 10.

that the positive sign applies on one side of the battery, the negative on the other. $E_0$ is the voltage at a point chosen as the origin. One need not choose the battery position as the origin: the origin may be taken on either side of the battery for either branch of the curve and $E_0$ will be the extrapolated value of the curve if the origin is beyond that branch of the curve.

It follows from Eqs. 10 and 11 that

$$\frac{de/dx}{e} = \pm\alpha;$$

$$\frac{di/dx}{i} = \pm\alpha.$$

That is, the per unit (angular) change in the variable per unit distance along the line is constant, equal to $\pm\alpha$; and, therefore, a distance $x$ along the line represents an angle (per unit change) $\pm x\alpha$.

This is the physical meaning of angle when a length of line is spoken of as so many radians.

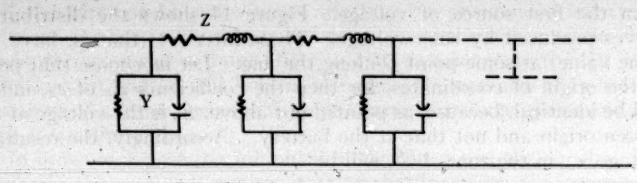


FIG. 12. A leaky a-c. transmission line.

If the applied voltage is alternating, and we wish to deal with the effective vector values (time vectors) of $e$ and $i$, we must change $R$ to $Z$, and $G$ to $Y$ (Fig. 12), so that

$$\alpha = \pm \sqrt{ZY} = \pm \sqrt{(R + jX_L)(G + jB_C)}. \tag{12b}$$

If $R$ and $G$ are negligible compared with $X$ and $B$, then

$$\alpha = \pm j\sqrt{X_L B_C} \tag{12c}$$

and

$$e_x = E_0 \epsilon^{\pm j\sqrt{X_L B_C}\, x} \tag{13}$$

$$i_x = I_0 \epsilon^{\pm j\sqrt{X_L B_C}\, x}. \tag{14}$$

In Eqs. 13 and 14, $e$ and $i$ and $E_0$ and $I_0$ are all effective values of time vectors, and therefore we will not have any occasion to speak of anything like sin $wt$.

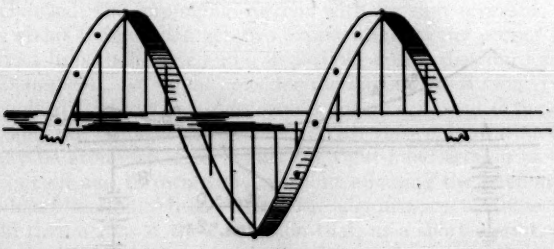

FIG. 13. Alternating current and voltage distribution in a loss-free line of unlimited length, or one equivalent to unlimited length as in surge-impedance loading of a line. The vectors are of constant magnitude but with continuously changing phase along the line.

Equations 13 and 14 mean that the voltage and current vectors are constant in magnitude but change their phases as we go along the line, like the radii of a cork screw (Fig. 13).

There is no thought of either a circular or a hyperbolic sine or cosine in the foregoing telegraph line equations or in the a-c. line equations for a line of unlimited length.

Keeping the lines still unlimited in length, let us connect a second source of voltage of arbitrary magnitude to the line at some distance from the first source of voltage. Figure 14 shows the distribution curves produced by each voltage. These intersect (that is, have the same value) at some point $O$ along the line. Let us choose that point as the origin of coordinates, for then the coefficients $E_0$ of $e_A$ and $e_B$ will be identical, because, as pointed out above, $E_0$ is the voltage at the chosen origin and not that at the battery. Accordingly, the resultant voltage, $e_x$, in the zone $A$–$B$, will be

$$\begin{aligned} e_x &= e_A + e_B \\ &= E_0\epsilon^{-ax} + E_0\epsilon^{ax} \\ &= E_0(\epsilon^{+ax} + \epsilon^{-ax}). \end{aligned} \tag{15}$$

Obviously, at $x = 0$, $e_x = 2E_0$. Suppose we replace $2E_0$ by the symbol $E$. Then

$$e_x = E\frac{\epsilon^{ax} + \epsilon^{-ax}}{2}. \tag{16}$$

The current equations will be similar except for the algebraic signs of the two components, because *they flow in opposite directions*. Thus,

$$i_x = i_A - i_B. \tag{17}$$

At $O$, the two currents will be equal and opposite, and the equation of the resultant current will take the form

$$i = I\frac{\epsilon^{+ax} - \epsilon^{-ax}}{2}. \tag{18}$$

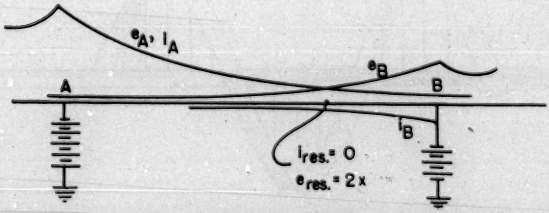


FIG. 14. The leaky telegraph line excited from two different points, giving rise to hyperbolic functions.

We observe the hyperbolic cosine and sine in Eqs. 16 and 18, and realize that they originate from the superposition of two excitations applied at different points on the line. Now we shall consider a very common way of bringing about the same situation without the benefit of a second source of voltage.

We have seen that, at $O$, the voltage is doubled and the resultant current in the wire is zero. Now in any circuit where the current is zero, the wire can be cut without affecting the phenomena anywhere; therefore, the line in Fig. 14 can be cut at $O$, and the system to the right can be thrown away without affecting the phenomena to the left of $O$. Equations 16 and 18 still hold in the zone $AO$, but now they are the equations of a finite line with a single source of voltage at one end and *open at the other end.*

In order to expose the full physical significance of the cut line, Fig. 15 shows the retained portion $A$–$O$ of the line, with the component currents and voltages in solid lines, and extrapolations of $e_A$, $i_A$, in dotted lines that happen to coincide. It may be observed that $e_B$ *is the mirror image of the dotted* $e_A$, *and* $i_B$ *is the reversed mirror image of the dotted* $i_A$.

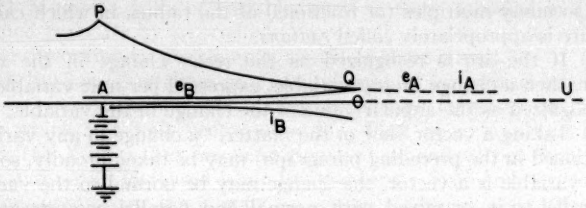


FIG. 15. The open-circuited leaky line giving rise to reflection and the same kind of current and voltage distributions as in Fig. 14.

Obviously, an open circuit reflects the impinging voltage with its sign unchanged, the impinging current with its sign reversed; and so it comes about that we can get two exponentials of the proper kind to constitute a hyperbolic sine and a hyperbolic cosine distribution.[4]

As Example II, we might consider the battery at $B$ (Fig. 14) with *reversed* polarity, in which case we would find that now at $O$ the current doubles and the voltages cancel out. With zero potential at $O$, that point may be grounded, after which the right-hand section of the line may be cut off and thrown away, without affecting the phenomena on the left-hand section. Interpreting the significance of these results, we would then arrive at the conclusion that, at a short-circuit, current is reflected with original sign, voltage with reversed sign, thus yielding a second exponential to match an impressed exponential distribution and produce a hyperbolic sine or cosine distribution.

For an Example III, we need merely hint now that starting with the spiralling vector a-c. voltage of Fig. 13, if we should impress a similar

[4] For those who have a previous concept about reflection from an open circuit, and find it difficult to reconcile it with the foregoing, the hint may be given that what is reflected is always what would normally have gone *past* the point of reflection. Not $PQ$ (Fig. 15) but $QU$ will be reflected. Otherwise, how can a thing that hasn't touched the reflecting point be reflected?

excitation, but with a negative spiral, at another point of the line, and explore along the line, we would find some point at which the current is zero and the conductor may be cut, or some point at which the potential is zero and therefore the line may be grounded, etc., ending up with the discovery that a-c. vectors also are reflected at an open or short-circuited point, yielding a reflected spiral vector distribution to match the impressed distribution in a way to produce a circular sine or cosine space distribution of the vector voltage and current along the line. Thus it is seen that reflection at an open circuit or short circuit provides a matching vector to an original vector to produce a circular or hyperbolic function.

SUMMARY

(*a*) An angle, as ordinarily visualized, is judged by its arc and is measured most logically by the per-unit-radius length of the arc, that is, as so many multiples (or fractions) of the radius, in which case the measure is appropriately called *radians*.

(*b*) If the arc is recognized as the *vector* change in the radius vector, then a change in any variable, expressed per-unit variable, will be recognized as the angular value of the change in the variable.

(*c*) Taking a vector view of the matter, "a change in any variable" (mentioned in the preceding paragraph) may be taken broadly, so that, if the variable is a vector, the change may be normal to the variable, or parallel to it, or mixed with normal and parallel components. A change normal to the variable, expressed per unit, constitutes a circular angle for obvious reasons. A change parallel to the variable, expressed per unit, constitutes a hyperbolic angle for unobvious conventional reasons. If the variable is a scalar, then its per unit measure of change will be called a hyperbolic angle. A change with both components, expressed per unit, then constitutes a complex angle.

Obviously the arcs of a circular angle and a hyperbolic angle are perpendicular to each other. Using complex numbers, and treating the radius vector as the reference axis, circular radians will be imaginary ($j$) numbers, hyperbolic radians real numbers.

(*d*) All per unit reckoning is understood to be like compound percentage reckoning. This implies that each element of the arc, normal to the radius vector, will be normal, not to the initial radius vector $\rho_0$ (Fig. 6), but to the radius vector drawn to that element. That is the vector reason why the arc takes the shape of a circle. Also, the per unit value of each element of the arc parallel to the radius vector will have to be expressed not as a fraction of the initial value of the variable but of the value of the variable just before that element of change has taken place. This is the reason why an element of arc parallel to the radius vector and having a given absolute length will have a different angular value depending on its position along the arc.

(*e*) An angle may also be specified by the two radii vectors, $\rho_2$

and $\rho_1$, bounding it. In this case, the compound ratio of $\rho_2$ to $\rho_1$, as $\log(\rho_2/\rho_1)$, yields the same measure as the corresponding arc in radians, for any arbitrary values of $\rho_2$ and $\rho_1$. This then offers a simple key to the logarithms of negative, imaginary and complex numbers. Thus, this theory makes it obvious that $\log(-1)$ is $\pi j$; $\log j = (\pi/2)j$; etc.

(*f*) The resultant of a variable, undergone a positive per unit change, and a like variable, undergone an equal negative per unit change, is the cosine of the per unit change if the change is normal to the variable; hyperbolic cosine, if the change is parallel to the variable. Similarly, the resultant of a variable, undergone a positive per unit change, and a like but negative one undergone an equal negative per unit change, is the sine of the per unit change, if the change is normal to the variable; and hyperbolic sine, if the change is parallel to the variable.

(*g*) Matching pairs of variables such as those specified in (*f*) are obtained generally by an original (impressed) variable and its reflection from a discontinuity like an open circuit or short circuit.

(*h*) Epsilon ($\epsilon$) qualifies as a natural base for logarithms because its various powers mean corresponding percentage increases on its base value which is a unit vector ($\epsilon^0 = 1$). That is, $\epsilon^n$ is a unit vector (along the $X$-axis) that has been increased $n$-hundred per cent, and $\epsilon^1$ is the unit vector increased 1-hundred per cent. Such an interpretation does not apply to $a^n$ if $a$ is not $\epsilon$.

**Music-Composing Machine.***—A music-composing machine could be constructed on the following principles:

Since all musical compositions are expressed in a musical notation, composed of a small number of symbols, permutations and combinations of these symbols would produce all possible musical compositions.

Let a machine be constructed for the rapid permuting and combining of musical symbols. Such machine would probably embody engineering principles utilized in the construction and operation of high-grade calculating machines.

Musicologists would select all material produced by such a machine having musical value.

The totality of such selected musical material would constitute the entire realm of creative music.

It is believed that the above proposal is thoroughly practicable, and is called to the attention of musical machine specialists.

**Anticorrosion liquid may be pumped moisture-free** into aircraft engines being prepared for storage, through use of a method invented by an Air Force sergeant in Japan. Reported to save 1000 man-hours a week, it consists of a compressor which draws air through a dehydrator, then pumps it into an oxygen tank which has been filled with the anticorrosion liquid. An engine may be filled by removing a spark plug and inserting a hose with a triggered nozzle, releasing the liquid under pressure.

**A saving of nearly 900 per cent in man-hours** required to recable a 100-ton, 100-foot stiff-leg crane has been achieved through a plan used at the San Francisco Port of Embarkation. Seven men, instead of 8 to 10 as in older methods, spliced the new five-ton cable to one end of the old cable. The old cable was then pulled out of the sheaves by means of a tractor, and onto a spare cable-reel hookup. The job was completed in nine hours, whereas an ordinary hand replacement operation takes two weeks.

**A photoelectric fire-detection device for aircraft use** has been developed and is under study for possible use by the Air Force. Any kind of flame, but not bright sunlight, will set off the detection unit. In recent tests, a gasoline fire kindled in a 2½-inch pan in direct sunlight was immediately detected. Once the flame was extinguished, all indication stopped.

**A collapsible 26-ft. dory with a 3000-lb. cargo capacity,** redesigned from the Eskimo "Umiak" for the Army Transportation Corps, will be given extensive tests this winter in Arctic waters. The 500-foot craft consists of a frame divided into four sections, over which fits a 60-lb. rubber-impregnated nylon skin. Packed in the stern section is a 10-hp. outboard motor which produces speeds up to 8 mph. The "Umiak" may be portaged by five men and quickly assembled. The Army hopes to find it feasible to drop the sections by parachute.

* This proposal was submitted to the JOURNAL by Dr. Armand Lowinger, 311 Carlton Avenue, Brooklyn, N. Y.

# FLUCTUATIONS IN PROPORTIONAL COUNTERS *

BY

W. F. G. SWANN [1]

It has been found by P. Weisz and the present author that a cosmic ray traversing similar path lengths in two proportional counters gives pulses $A$ and $B$ which are unequal, $A$ being sometimes less than and sometimes greater than $B$. The path length was 20 cm. of mercury in argon at a total pressure of 40 cm. of Hg. and was such as to give about 300 primary events. This number is far too large to provide an explanation of the phenomenon on the basis of statistical fluctuations in the number of events themselves, and we must look for something more deepseated.

The primary events are the emissions of secondary electrons which we shall call "primary secondaries." Each of these proceeds to produce others, and the ultimate number of ions responsible for initiating avalanches in the counter is the total number of ions so produced.

Now according to theory, the average number of primary secondaries emitted with energies between $Q$ and $Q + dQ$ is

$$dN = A \frac{dQ}{Q^2}, \tag{1}$$

with a lower limit $Q_m \sim 10$ e.v. and an upper limit $Q_M \sim 10^8$ e.v., $A$ being a constant. Fortunately, the phenomena to be discussed are not very sensitive to the exact magnitude of $Q_M$. Thus, the total number of primary secondaries is $N$, where

$$N = A \int_{Q_m}^{Q_M} \frac{dQ}{Q^2} = A \left( \frac{1}{Q_m} - \frac{1}{Q_M} \right) = \frac{A}{Q_m} \text{ approx.}$$

Hence

$$A = NQ_m$$

and

$$dN = NQ_m \frac{dQ}{Q^2}. \tag{2}$$

* The present communication is presented at this time as pertaining to work sponsored by the Joint Program of the Office of Naval Research and the Atomic Energy Commission. Papers on the matter have been presented by the present author before the American Physical Society (Baltimore, May 1–2, 1942; Cambridge, April 25–27, 1946). Since, however, the mathematical details given in the papers were not given in the abstracts (*Phys. Rev.*, Vol. 61, p. 734 (1942); *ibid.*, Vol. 69, p. 690 (1946), the present paper is written with the object of supplying such details.

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The number of ions ultimately attributable to a primary secondary is proportional to $Q$, and it is of interest to ascertain the value $Q_o$ of $Q$ such that half of all the ionization is produced by primary secondaries whose energy is greater than $Q_o$. The value of $Q_o$ is given by

$$NQ_m\int_{Q_m}^{Q_o} Q\frac{dQ}{Q^2} = NQ_m\int_{Q_o}^{Q_M} Q\frac{dQ}{Q^2}$$

$$\log\frac{Q_o}{Q_m} = \log\frac{Q_M}{Q_o} \tag{3}$$

$$Q_o^2 = Q_mQ_M. \tag{4}$$

Putting $Q_m = 10$ and $Q_M = 10^8$, we find $Q_o = 3.2 \times 10^4$ e.v. Thus half the total ionization is produced by the very few primary secondaries which have energy greater than $3.2 \times 10^4$ e.v. Naturally, the statistical fluctuation in these very few high energy primary secondaries is very great and carries with it great fluctuations in the ionization as measured.

The fraction $P$ of the $N$ primary secondaries, with $Q > Q_o$, is

$$P = \int_{Q_o}^{Q_M}\frac{dQ}{Q^2}\Big/\int_{Q_m}^{Q_M}\frac{dQ}{Q^2}$$

$$= \left(\frac{1}{Q_o} - \frac{1}{Q_M}\right)\Big/\left(\frac{1}{Q_m} - \frac{1}{Q_M}\right) = \frac{Q_m}{Q_o} \text{ approx.} \tag{5}$$

Hence $P = 10/3.2 \times 10^4 = 3.2 \times 10^{-4}$.

The average number of cases where $Q > Q_o$ in the passage of a primary through the counter along the path designated is

$$PN = 300 \times 3.2 \times 10^{-4} = 0.1.$$

Now if, as a first approximation, we assume that the number of primary secondaries for $Q > Q_o$ shows fluctuations in accordance with Poisson's law, so that the fraction $F$ of the primary rays which give $n$ primary secondaries is

$$F = \frac{x^n}{\underline{n}}e^{-x},$$

where $x$ is the average number of primary secondaries per particle, we have $x = 0.1$. The values of $F$ for $n = 0, 1, 2$, respectively are 0.9, 0.09, 0.005. Thus, in 90 per cent of the cases, phenomena which, on the average, contribute 50 per cent to the total ionization are absent.

Suppose now we should ask for the value $Q_1$ of $Q$ for which 60 per cent of the ionization is produced by primary secondaries with $Q > Q_1$. Equation 3 becomes replaced by

$$\log\frac{Q_1}{Q_m} = \frac{4}{6}\log\frac{Q_M}{Q_1} = \log\left(\frac{Q_M}{Q_1}\right)^{2/3},$$

$$Q_1 = Q_m^{3/5}Q_M^{2/5},$$

$$Q_1 = 10^4/1.6.$$

For this case (5) becomes replaced by

$$P = \frac{Q_m}{Q_1} = 1.6 \times 10^{-2}$$

and

$$PN = 300P = 0.48,$$

and from Poisson's formula we find for $n = 0, 1, 2$, respectively, the values 0.62, 0.30, 0.07, so that, in 62 per cent of the cases of passage of a primary through the counter, events are missing which constitute 60 per cent of the total average ionization.

The situation raised by the foregoing considerations is, of course, but further complicated by the fact that the dimensions of the counter will, in general, limit the range of action of primary secondaries of large energy.

*Further Considerations*

The total energy in all the ions produced by the primaries and secondaries is $W$, where

$$W = \int_{Q_m}^{Q_M} Q dN = NQ_m \int_{Q_m}^{Q_M} \frac{dQ}{Q} = NQ_m \log \frac{Q_M}{Q_m}. \tag{6}$$

If $n$ is the average total number of ions produced per centimeter of path, $l$ the path length, $n_p$ the number of primary secondaries emitted per centimeter of path, $e$ the energy per ion, we have

$$nel = ln_pQ_m \log \frac{Q_M}{Q_m} \tag{7}$$

$$\frac{n}{n_p} = \frac{Q_m}{e} \log \frac{Q_M}{Q_m}. \tag{8}$$

If $Q_m = e = 10$ e.v., and $Q_M = 10^8$ e.v.

$$\frac{n}{n_p} = 16$$

which is larger than it should be. However, $n/n_p$ is quite sensitive to the exact form of (1). Thus, if we should alter (1) to

$$dN = A \frac{dQ}{Q^{2+\epsilon}}$$

we should have

$$N = \frac{A}{1+\epsilon}\left[\frac{1}{Q_m^{1+\epsilon}} - \frac{1}{Q_M^{1+\epsilon}}\right] = \frac{A}{(1+\epsilon)Q_m^{1+\epsilon}} \text{ approx.}$$

so that (2) becomes replaced by

$$dN = N(1+\epsilon)Q_m^{1+\epsilon}\frac{dQ}{Q^{2+\epsilon}}.$$

Equation 6 becomes replaced by

$$W = N\frac{(1+\epsilon)}{\epsilon}Q_m^{1+\epsilon}\left[\frac{1}{Q_m^{\epsilon}} - \frac{1}{Q_M^{\epsilon}}\right]$$

$$= N\left(\frac{1+\epsilon}{\epsilon}\right)Q_m\left[1 - \left(\frac{Q_m}{Q_M}\right)^{\epsilon}\right]$$

and (8) becomes replaced by

$$\frac{n}{n_p} = \left(\frac{1+\epsilon}{\epsilon}\right)\frac{Q_m}{e}\left[1 - \left(\frac{Q_m}{Q_M}\right)^{\epsilon}\right].$$

Thus, if $\epsilon = 0.2$, $Q_m = e = 10$ e.v., $Q_M = 10^8$ e.v.

$$\frac{n}{n_p} = 6,$$

which approximates a more reasonable value.

On the other hand, (3) becomes changed to

$$\frac{1}{Q_m^{\epsilon}} - \frac{1}{Q_o^{\epsilon}} = \frac{1}{Q_o^{\epsilon}} - \frac{1}{Q_M^{\epsilon}} \tag{9}$$

$$Q_o = \frac{2^{1/\epsilon}Q_m}{\left[1 + \left(\frac{Q_m}{Q_M}\right)^{\epsilon}\right]^{1/\epsilon}}$$

so that, if $Q_m = 10$, $Q_M = 10^8$ and $\epsilon = 0.2$

$$Q_o = 262.$$

Moreover, (5) becomes changed to

$$P = \left(\frac{Q_m}{Q_o}\right)^{1+\epsilon} \text{ approximately} \tag{10}$$

$$= \left(\frac{1}{26.2}\right)^{1.2} = 0.02$$

$$PN = 300 \times 0.02 = 6.$$

The value of $F$ for the case $n = 0$ is now negligible instead of being 0.9.

On the other hand, if we ask for the value $Q_1$ of $Q$ for which 25 per cent of the ionization is produced by primary secondaries with $Q > Q_1$,

we have, by suitable modification of (9)

$$\frac{1}{Q_m{}^\epsilon} - \frac{1}{Q_1{}^\epsilon} = 3\left(\frac{1}{Q_1{}^\epsilon} - \frac{1}{Q_M{}^\epsilon}\right).$$

$$Q_1 = \frac{4^{1/\epsilon} Q_m}{\left[1 + 3\left(\frac{Q_m}{Q_M}\right)^\epsilon\right]^{1/\epsilon}} = 5800.$$

Moreover, (5) becomes changed to

$$P = \left(\frac{Q_m}{Q_1}\right)^{1+\epsilon} \text{ approx.}$$

so that

$$P = \left(\frac{1}{580}\right)^{1.2} = 0.48 \times 10^{-3}.$$

Hence, with $N = 300$, $PN = 0.14$, which leads to the conclusion that in a fraction $e^{-0.14}$, that is, in 87 per cent of the passages of a primary ray through the counter, events responsible for 25 per cent of the ionization will be missing.

While, therefore, the foregoing calculations serve to show the sharp dependence of the statistical effects upon the exact form of the law exemplified in (1), they show the vital importance of such effects on the interpretation of experimental data.

While the foregoing considerations have been developed with cosmic rays as the primaries in mind, there is every reason to expect similar considerations applicable to nuclear particles of much smaller energy, with energies, in fact, comparable with those of $\beta$ rays.

**Atomic Energy for Aircraft.**—Atomic energy may some day propel supersonic bombers nonstop around the world, a scientist from Oak Ridge, Tennessee, recently told attendants at the National Conference on Industrial Hydraulics.

"Atomic energy as a source of power for aircraft promises airplanes combining extremely high speed with almost unlimited range," Andrew Kalitinsky, Chief Engineer of the NEPA Project, a Division of Fairchild Engine and Airplane Corporation, declared in an address following a banquet of the Conference.

Kalitinsky pointed out that atomic power offers fuels such as uranium that would not be consumed nearly so fast as those in chemically fueled planes. He named high operating temperatures, heat transfer, and weight of the mass needed to stop radiations as some of the problems encountered.

"Developing a nuclear aircraft power plant will be difficult," Kalitinsky said, "but there are several basic types of power plants that can be adapted to utilize this power. All are thermal power plants, since fission energy is released predominantly in the form of heat. Examples would be turbo-props, turbo-jets, ram-jets, and rockets."

Because chemically fueled planes must be made larger and heavier in order to increase speed and range, there is a performance limit imposed. On the other hand, atomic fuel of almost constant size and weight and high heating value offers attractive possibilities, Kalitinsky told the hydraulics experts.

"Our present supersonic airplanes run out of fuel in a matter of a few minutes. With atomic power, they could keep going because the fuel supply would remain nearly constant," he said.

"The engine component of the nuclear installation may be expected to weigh about the same as a conventional power plant of the same horsepower and thrust rating. Weight of the reactor shielding would be approximately equivalent to the weight of fuel and fuel tanks in large chemically powered aircraft."

Transfer of heat generated in the reactor to the working fluid, protection of the radioactive material against corrosion by the working fluid (or the escape of fission products from the reactor into the working fluid), and the choice of shielding materials, including the consideration of density and resultant decrease of volume, are among development problems cited by the speaker.

"Atomic aircraft will have to be designed to operate at very high speeds to take full advantage of atomic power," Kalitinsky predicted. "It will be a large airplane, and it will be designed for an extremely high landing weight since practically no fuel will be consumed during flight. Crew members will be placed as far as possible from the nuclear reactor to protect them from harmful radiations."

# THEORETICAL LIMITATIONS ON THE BROADBAND MATCHING OF ARBITRARY IMPEDANCES *

BY

R. M. FANO [1]

## 4. THE DESIGN OF SIMPLE MATCHING NETWORKS

The integral relations derived above indicate that the ideal type of behavior for the return loss at the input terminals of a matching network is represented by the rectangular-shaped function used in the determination of the optimum tolerance. Such a behavior cannot be obtained in practice because it requires a matching network with an infinite number of elements, but can be approximated sufficiently well for practical purposes by means of a reasonably small number of elements. In other words, the function representing $|\rho_1|$ must be selected in such a way as to approximate a constant over the pass band and unity over the attenuation band, just as in the case of conventional filters. It must be pointed out, however, that filters are designed in most cases to provide a perfect match at a number of frequencies in the pass band, while such a situation is to be avoided in the case of matching networks. In fact, making $\ln 1/|\rho_1|$ very large at any point of the pass band leads to an inefficient use of the areas represented by the integrals discussed above, and results, therefore, in a reduction of the bandwidth of approximate match. In spite of this essential difference between the characteristics of filters and matching networks, the same techniques can be used in both cases for the solution of the approximation problem. This point is made clear by the illustrative examples discussed below.

A very simple and important type of matching problem is presented by the case of a load impedance consisting of a resistance in series with an inductance, or by the dual case of a resistance shunted by a condenser. Practical problems of this type arise, for instance, in connection with the broadbanding of the high-frequency response of matching transformers, or when a resistive load is shunted by a stray capacitance. A method of designing appropriate matching networks for a series $RL$ impedance is developed below. The same method will be directly applicable to the dual case of a shunt $RC$ admittance.

The pass band desired in most of these problems extends from zero frequency to some cut-off frequency $\omega_c$; the ideal behavior for the return loss is, therefore, that illustrated in Fig. 6. Let $L_1$ be the value of the inductance normalized with respect to the series resistance, that

* Continued from the January issue of the Journal, p. 57.

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is divided by it. The coefficient $A_1^\infty$ is, by definition,

$$A_1^\infty = \left[ \frac{d}{d\frac{1}{\lambda}} \left( \ln \frac{2+\lambda L_1}{\lambda L_1} \right) \right]_{\frac{1}{\lambda}=0} = \frac{2}{L_1}. \tag{31}$$

The maximum theoretical pass-band value of $\ln 1/|\rho_1|$ is, therefore, according to Eq. 24

$$(\ln I/|\rho_1|)_{\max} = \frac{\pi}{\omega_c L_1}. \tag{33}$$

The problem consists, then, of approaching this theoretical limit by means of a matching network involving a finite and prescribed number of elements.

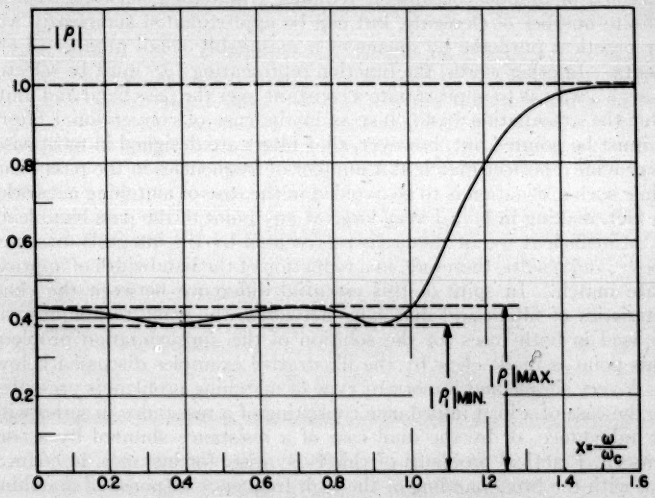


FIG. 16. Typical frequency behavior of $\rho_1$ with Tchebysheff approximation (computed for the network of Fig. II-1).

The general remarks made above indicate that the inductance $L_1$, which forms the network $N'$, may be considered as the first element of a low-pass filter, the network $N$, whose input reflection coefficient is $\rho_1$. This reflection coefficient cannot be measured in practice, because the inductance $L_1$ is inseparable from the resistive termination; its magnitude, however, is equal to the magnitude of the reflection coefficient $\rho_2$ at the other terminals of the filter to which the generator will be connected in actual operation.

Two types of functions are used for the solution of the approximation problem in the case of low-pass filters (3, 8).[2] The first type of function is the Tchebysheff polynomial $T_n(\omega/\omega_c)$ which leads to a function $|\rho_1|$ which oscillates between two given values in the pass band, and asymptotically approaches unity in the attenuation band, as illustrated in Fig. 16. The second type of function is the Jacobian elliptic function which leads to an oscillatory behavior of $|\rho_1|$ in both the pass band and the attenuation band. In the first case, all the zeros of transmission are at infinity so that the network consists of a simple ladder structure with a total of $n$ series inductances and shunt condensers. In the second case, zeros of transmission are present at finite frequencies as well as at infinity, and the resulting network has a form similar to the "$m$-derived" filters. The design involving elliptic functions leads to a slightly better tolerance, but the difference does not seem to be worth the resulting theoretical and practical complications.

The Tchebysheff polynomial of the first kind and order $n$ is defined by

$$T_n(x) = \cos(n \cos^{-1} x).$$

It is clear that this function oscillates between plus and minus one for $|x| < 1$, and approaches $\pm \infty$ for $|x| > 1$. In order to obtain the function $|\rho_1|^2$, one first constructs a function $|t|^2$ which has oscillatory behavior in the pass band, is smaller than unity, and has all its zeros at infinity. Letting $x = \omega/\omega_c$, one obtains

$$1/|t|^2 = (1 + K^2) + \epsilon^2 T_n^2(x),$$

where $K$ and $\epsilon$ are arbitrary constants. The corresponding magnitude of the reflection coefficient becomes, after appropriate manipulation,

$$|\rho_1|^2 = 1 - |t|^2 = [\rho_1(z)\rho_1(-z)]_{z=jx}$$

$$= \begin{cases} \left[\dfrac{\cosh n(\sinh^{-1} z - b)\cosh n(\sinh^{-1} z + b)}{\cosh n(\sinh^{-1} z - a)\cosh n(\sinh^{-1} z + a)}\right]_{z=jx} & (n \text{ even}) \\ \left[\dfrac{\sinh n(\sinh^{-1} z - b)\sinh n(\sinh^{-1} z + b)}{\sinh n(\sinh^{-1} z - a)\sinh n(\sinh^{-1} z + a)}\right]_{z=jx} & (n \text{ odd}) \end{cases} \tag{34}$$

where $z = \lambda/\omega_c$, $\sinh^2 nb = (K/\epsilon)^2$, and $\sinh^2 na = (1 + K^2)/\epsilon^2$. The poles of this function are evidently given by

$$z_p = \begin{cases} \sinh\left[\pm a \pm j\dfrac{\pi}{n}(m + \tfrac{1}{2})\right] & (n \text{ even}), \\ \sinh\left[\pm a \pm j\dfrac{\pi}{n} m\right] & (n \text{ odd}), \end{cases} \tag{35}$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

where $m$ is an integer or zero. The zeros are given by the same expression in which $b$ is substituted for $a$. It will be noted that the poles lie on an ellipse centered at the origin with semiaxes equal to cosh $a$ and sinh $a$, as indicated in Fig. 17 for the two cases of $n = 3$ and $n = 4$. The zeros lie similarly on an ellipse of semiaxes equal to cosh $b$ and sinh $b$.

The poles of $\rho_1$ are necessarily those poles of $[\rho_1(\lambda)\rho_1(-\lambda)]$ which lie in the left half-plane; that is, $a$ must be taken with the negative

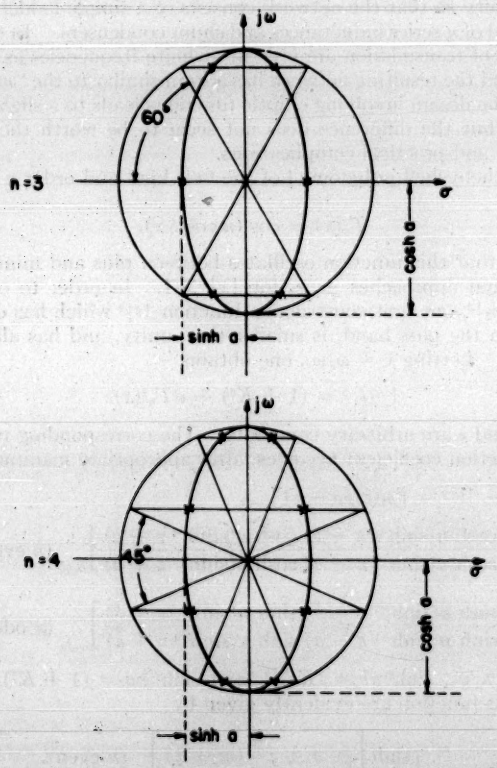


FIG. 17. Location of the poles of $\rho_1(\lambda)\rho_1(-\lambda)$ for a network with three or four elements.

sign. The zeros of $\rho_1$, on the contrary, can be located anywhere in the complex plane, as far as the physical realizability of the function is concerned. It has been shown before, however, that the area represented by $\int_0^\infty \ln 1/|\rho_1|\, d\omega$ is a maximum for a given $A_1^\infty$ when all the zeros of $\rho_1$ are in the left half-plane. Therefore $b$ must be taken also

with the negative sign. The coefficient $A_1^\infty$ corresponding to the resulting function $\rho_1$ is then computed by means of Eq. 10 as follows

$$A_1^\infty = \sum_i (\lambda_{0i} - \lambda_{pi}) = \omega_c \frac{\sinh a - \sinh b}{\sin \pi/2n}. \tag{36}$$

The quantities $a$ and $b$ must satisfy this equation since $A_1^\infty$ is specified by the load impedance through Eq. 32. In addition, $a$ and $b$ must be

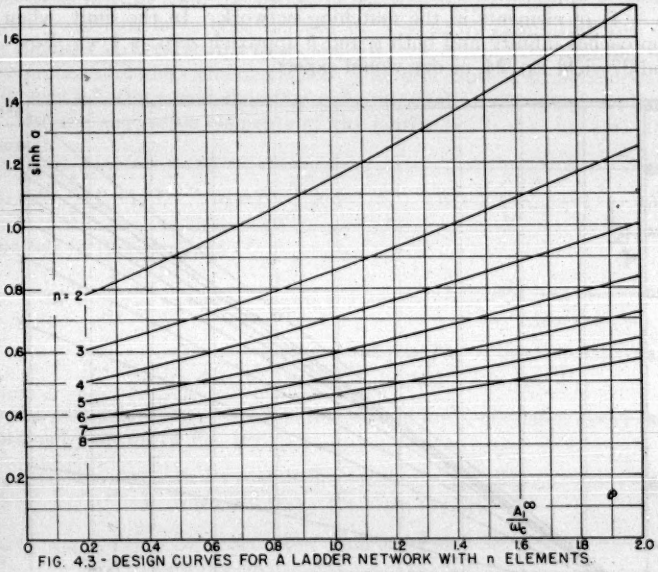


FIG. 18. Design curves for a ladder network with $n$ elements.

chosen so as to minimize the tolerance of match. One obtains from Eq. 34

$$|\rho_1|_{\max} = \frac{\cosh nb}{\cosh na}. \tag{37}$$

This value is then minimized, subject to the condition imposed by Eq. 36, by using the method of indeterminate multipliers. The result is

$$\frac{\tanh na}{\cosh a} = \frac{\tanh nb}{\cosh b}. \tag{38}$$

The parameters $a$ and $b$ are determined, finally, by solving simultaneously Eqs. 36 and 38. The resulting optimum value of $\sinh a$ is plotted in Fig. 18 as a function of $A_1^\infty/\omega_c$ for different values of $n$ up to and

including 8.[3] The corresponding values of $\sinh b$ can be obtained by means of Eq. 36.

The optimum value of the tolerance can be determined by means of Eq. 37. Figure 19 presents a plot of $\ln 1/|\rho_1|_{max}$ as a function of $A_1^\infty/\omega_c$ for different values of $n$. The curve indicated with $n = \infty$ is the straight line of slope equal to $\pi/2$ which represents the limiting value of the tolerance given by Eq. 24. It will be noted that this limiting value is approached reasonably well with a relatively small number $(n - 1)$ of elements in the matching network. In the limit, when $n$ approaches infinity and both $a$ and $b$ approach zero as $1/\sqrt{n}$, Eqs. 36 and 37 yield Eq. 24, as one would expect.

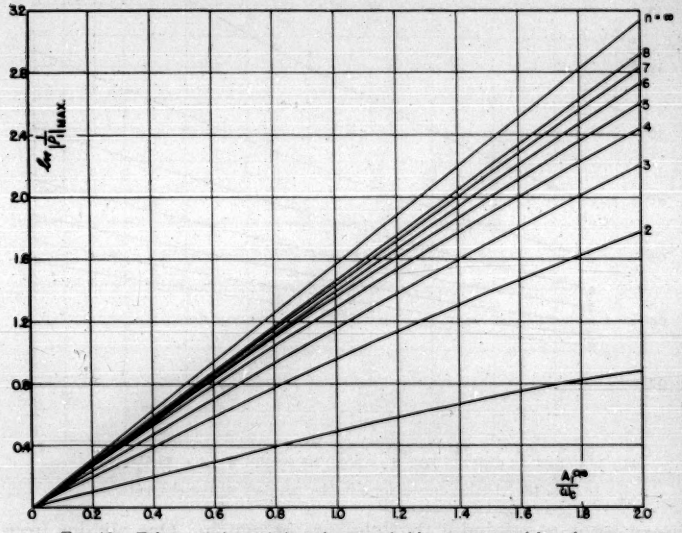


FIG. 19. Tolerance of match for a low-pass ladder structure with $n$ elements.

After determining the function $\rho_1$, one can compute, finally, the values of the elements of the matching network. For this purpose, any one of the available synthesis procedures can be followed, a discussion of which is beyond the scope of this paper (3, 8). It seems appropriate, on the other hand, to mention a method of computing the element values developed by the author in connection with the matching problem. This method has the advantage of permitting direct and independent computation of the individual elements from the values of $a$ and $b$ determined above.

[3] These curves are obtained from computations made by Dr. M. Cerrillo, following a graphical procedure suggested by Dr. E. A. Guillemin.

Consider a ladder structure consisting of series inductances and shunt condensers. The $A^\infty$ coefficients can be computed from the poles and zeros of $\rho_1$ by means of Eq. 10. On the other hand, the coefficient $A^\infty_{2k-1}$ depends only on the first $k$ elements of the ladder, so that the value of the $k^{\text{th}}$ element depends only on the coefficients with subscript smaller than or equal to $2k-1$. It should be possible, therefore, to compute the values of the elements directly from the $A^\infty$ coefficients, and these in turn from the values of the parameters $a$ and $b$. Appropriate equations have been derived for $k=4$, by computing the $A^\infty$ for a ladder structure with four elements, and solving the resulting set of equations for the element values. The procedure is straightforward but very laborious, and therefore only the final results are given here.

Let the successive elements of the ladder be $L_1$, $C_2$, $L_3$, etc., and also let

$$\alpha_3 = 2^2\frac{A^\infty_3}{(A_1)^3} - 1/3, \qquad \alpha_5 = 2^4\frac{A^\infty_5}{(A^\infty_1)^5} - 1/5, \qquad \alpha_7 = 2^6\frac{A^\infty_7}{(A^\infty_1)^7} - 1/7. \quad (39)$$

One has for the elements

$$L_1 = \frac{2}{A^\infty_1}, \qquad C_2 = -\frac{L_1}{\alpha_3}, \qquad L_3 = -\frac{\alpha_3 L_1}{1+\alpha_3-(\alpha_5/\alpha_3)},$$
$$C_4 = \frac{[1+\alpha_3-(\alpha_5/\alpha_3)]^2 L_1}{\alpha_3[1+\alpha_3-(\alpha_5/\alpha_3)+(\alpha_5/\alpha_3)^2-(\alpha_7/\alpha_3)]}. \quad (40)$$

In the particular case of the functional form for $\rho_1$ discussed above, one obtains by means of Eq. 10

$$A^\infty_3 = -2^{-2}\omega_c{}^3\left(\frac{\sinh 3a - \sinh 3b}{3\sin(3\pi/2n)} + \frac{\sinh a - \sinh b}{\sin(\pi/2n)}\right), \quad (41)$$

$$A^\infty_5 = 2^{-4}\omega_c{}^5\left(\frac{\sinh 5a - \sinh 5b}{5\sin(5\pi/2n)} + \frac{\sinh 3a - \sinh 3b}{\sin(3\pi/2n)} + 2\frac{\sinh a - \sinh b}{\sin(\pi/2n)}\right), \quad (42)$$

$$A^\infty_7 = -2^{-6}\omega_c{}^7\left(\frac{\sinh 7a - \sinh 7b}{7\sin(7\pi/2n)} + \frac{\sinh 5a - \sinh 5b}{\sin(5\pi/2n)} + 3\frac{\sinh 3a - \sinh 3b}{\sin(3\pi/2n)} + 5\frac{\sinh a - \sinh b}{\sin(\pi/2n)}\right). \quad (43)$$

It will be noted that the equations given above are sufficient for the design of a structure with eight elements. In fact four elements can be computed by operating from one end of the network and the other four by operating from the other end. The reflection coefficient $\rho_2$ which must be used in the second part of the design can be obtained from $\rho_1$ in a simple manner, as indicated in Appendix I. One must

keep in mind, however, that the network will, in general, involve an ideal transformer, since both terminations are assumed to be equal to one ohm. The turns ratio of the transformer can be determined easily from the zero-frequency behavior of the network. Illustrative examples are presented in Appendix II.

Next consider the design of a matching network for a load impedance consisting of a capacitance shunting a series *RL* combination. A problem of this type may arise in connection with the high-frequency response of step-up transformers, as discussed in Appendix III. The limiting tolerance for this matching problem is given by the curves in Fig. 7. This optimum tolerance was obtained by introducing a zero of $\rho_1$ at a point $\sigma_r$ of the positive real axis together with a symmetrical pole on the negative real axis. The same technique is used in obtaining the appropriate function for $\rho_1$ when the matching network must contain a specified number of elements. Also the same approximation function can be used for $|\rho_1|$ as in the case discussed above, because the addition of a zero and a pole symmetrically located with respect to the imaginary axis leaves the value of $|\rho_1|$ unchanged for imaginary values of $\lambda$. One obtains, in this case, from the conditions of physical realizability for $\rho_1$,

$$A_1^\infty/\omega_c = 2/\omega_c L_1 = \frac{\sinh a - \sinh b}{\sin(\pi/2n)} + 2\sigma_r/\omega_c, \tag{44}$$

$$-A_3^\infty/\omega_c^3 = 2^{-2}(A_1^\infty/\omega_c)^3(L_1/C_2 + 1/3) =$$

$$= 2^{-2}\left(\frac{\sinh 3a - \sinh 3b}{3\sin(3\pi/2n)} + \frac{\sinh a - \sinh b}{\sin(\pi/2n)}\right) - 2/3(\sigma_r/\omega_c)^3. \tag{45}$$

The maximum pass-band value of $|\rho_1|$ is still given by

$$|\rho_1|_{\max} = \frac{\cosh nb}{\cosh na}. \tag{46}$$

The parameters $a$, $b$, and $\sigma_r$ must be determined in such a way as to minimize the value of $|\rho_1|_{\max}$ and satisfy, at the same time, the equations above. This minimization process involves the solution of a system of transcendental equations. No convenient graphical procedure could be developed in this case.

Once $a$, $b$, and $\sigma_r$ have been determined, the values of the elements can be computed by following a procedure very similar to the one discussed above, but in this case a Darlington section (3) of type C (see Fig. III-2) is present in addition because of the zero of transmission at $\lambda = \sigma_r$. An illustrative design is carried out in Appendix III.

The design of matching networks for impedances of a more complex nature than those considered above is hampered in most cases by mathematical difficulties which lead to laborious numerical and graphical computations. It must be said, however, that many matching

problems of practical interest are of the types discussed above, or can be reduced to these types by means of simple changes of the frequency variable. In addition, a rigorous method of design can at times be combined effectively with a cut-and-try procedure. For instance, the frequency behavior of a given load impedance might first be modified empirically in such a way as to approximate, over the desired frequency band, the behavior of a simpler impedance function for which a rigorous design procedure is available. In such cases the ingenuity of the designer becomes of primary importance, since the technique to be used may vary considerably from one type of problem to another.

*Acknowledgment*

I wish to express my deep and sincere gratitude to Professor E. A. Guillemin, who supervised this work, for his constant advice and encouragement. In addition, and above all, I owe to his inspired teaching the knowledge of network theory on which the entire work presented in this paper is based.

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(3) S. DARLINGTON, "Synthesis of Reactance 4-Poles," *Journal of Mathematics and Physics*, Vol. XVIII, pp. 275–353, September 1939. Also Bell Telephone System Monograph B-1186.

(4) H. W. BODE, "A Method of Impedance Correction," *B.S.T.J.*, Vol. IX, pp. 794–835, October 1930.

(5) E. A. GUILLEMIN, *op. cit.*, Ch. IX, Sec. 12.

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(7) H. W. BODE, *op. cit.*, Secs. 10.7 and 10.8.

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APPENDIX I

The reflection and transmission coefficients of a two-terminal-pair reactive network represent the characteristics of the network when one-ohm terminations are connected to both pairs of terminals, as

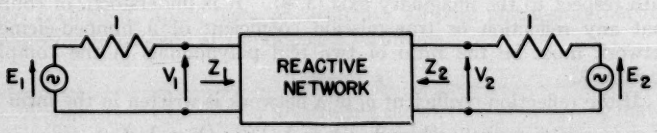



FIG. I-1. Two-terminal-pair reactive network with one-ohm terminations.

shown in Fig. I-1. The two reflection coefficients are defined by

$$\rho_1 = \frac{Z_1 - 1}{Z_1 + 1} = \left[\frac{2V_1}{E_1} - 1\right]_{E_2=0}, \tag{I-1}$$

$$\rho_2 = \frac{Z_2 - 1}{Z_2 + 1} = \left[\frac{2V_2}{E_2} - 1\right]_{E_1=0}, \tag{I-2}$$

where $Z_1$ and $Z_2$ are the impedances measured at the two pairs of terminals when the voltage sources are short-circuited. The transmission coefficient is defined, with reference to Fig. I-1, by

$$t = \left[\frac{2V_2}{E_1}\right]_{E_2=0} = \left[\frac{2V_1}{E_2}\right]_{E_1=0}. \tag{I-3}$$

The physical significance of these coefficients is best understood by inserting two transmission lines of unit characteristic impedance between the network and the terminations. The reflection coefficient $\rho_1$ is then the ratio of the voltage of the reflected wave to the voltage of the incident wave measured at terminals 1 for $E_2 = 0$; $\rho_2$ has the same significance for terminals 2. The transmission coefficient $t$ is the ratio of the voltage of the transmitted wave at terminals 2 to the voltage of the incident wave at terminals 1, for $E_2 = 0$. Because of the reciprocity theorem, the same value of $t$ is obtained for transmission in the opposite direction.

It is clear from the above definitions that $|\rho_1|^2$ is the per unit power reflected and $|t|^2$ is the per unit power transmitted. Since the network is non-dissipative, one obtains

$$|\rho_1|^2 = |\rho_2|^2 = 1 - |t|^2. \tag{I-4}$$

This equation implies that any function of the complex variable $\lambda = \sigma + j\omega$ which is to represent a reflection or transmission coefficient must have a magnitude smaller than, or equal to, unity at all points of the imaginary axis, that is at real frequencies. In addition, all the poles of this function must lie in the left half of the complex plane, because, otherwise, the network would oscillate upon any random disturbance. It can be shown (3, 8) that this condition on the poles together with the condition on the magnitude on the imaginary axis are sufficient as well as necessary conditions for the physical realizability of a reflection coefficient. In the case of a transmission coefficient, one must add the condition that the zeros be present in pairs symmetrical with respect to the imaginary axis (3, 8). It is understood, of course, that any reflection or transmission coefficient of a lumped-element network must be the ratio of two real polynomials in the complex variable $\lambda$.

If the reflection coefficient $\rho_1$ of a network is written in the form

$$\rho_1(\lambda) = K\frac{(\lambda - \lambda_{o1})(\lambda - \lambda_{o2}) \cdots (\lambda - \lambda_{on})}{(\lambda - \lambda_{p1})(\lambda - \lambda_{p2}) \cdots (\lambda - \lambda_{pn})}, \tag{I-5}$$

it can be shown that the reflection coefficient $\rho_2$ is given by

$$\rho_2(\lambda) = (-1)^{n+1}K\frac{(\lambda+\lambda_{o1})(\lambda+\lambda_{o2})\cdots(\lambda+\lambda_{on})}{(\lambda-\lambda_{p1})(\lambda-\lambda_{p2})\cdots(\lambda-\lambda_{pn})}. \quad \text{(I-6)}$$

The $\lambda_{oi}$ and the $\lambda_{pi}$ are, respectively, the zeros and poles of $\rho_1$; $K$ is a real constant.

A relation between the reflection coefficients and $t$ can be obtained by noting that

$$[\rho_1(\lambda)\rho_1(-\lambda)]_{\lambda=j\omega} = |\rho_1|^2_{\lambda=j\omega} = [\rho_2(\lambda)\rho_2(-\lambda)]_{\lambda=j\omega} = |\rho_2|^2_{\lambda=j\omega} \quad \text{(I-7)}$$

and

$$[t(\lambda)t(-\lambda)]_{\lambda=j\omega} = |t|^2_{\lambda=j\omega} = 1 - |\rho_1|^2_{\lambda=j\omega}. \quad \text{(I-8)}$$

It follows that the reflection and transmission coefficients have the same denominator, and therefore the same poles. The numerator of $t$ is either an even or an odd polynomial, because the zeros of $t$ must be present in pairs of quadruplets symmetrical with respect to the origin. It follows that the numerator of $[t(\lambda)t(-\lambda)]$ must be a perfect square; that is, its zeros must have even multiplicity. It must be pointed out in this regard that the function computed from either reflection coefficient by means of Eq. (I-8) might not satisfy this requirement. In such a case it is necessary to multiply both the numerator and the denominator of the function $[t(\lambda)t(-\lambda)]$ by the root factors of the numerator having odd multiplicity (3, 8). These root factors must be carried back into $\rho_1$ and $\rho_2$, in which they will appear after appropriate eliminations as pairs of zeros in the right half-plane together with symmetrical pairs of poles in the left half-plane. Such quadruplets of singularities do not change the magnitude of the reflection coefficient on the imaginary axis, but introduce only a phase shift. A corresponding elimination of zeros and poles in the left half-plane will take place in the transmission coefficient. In some cases $t$ contains a phase-shift factor, consisting of zeros in the right half-plane, and symmetrical poles in the left half-plane, and $\rho_2$ contains the same factor squared while such a factor is missing in $\rho_1$. This situation indicates that an all-pass network of unit characteristic impedance is connected at terminals 2 of the network so as to introduce a phase shift in both $t$ and $\rho_2$ without affecting $\rho_1$.

It can be concluded on the basis of the above discussion that a two-terminal-pair reactive network is completely specified by either reflection coefficient, apart from an arbitrary all-pass phase-shift network connected in cascade at the opposite terminals.

## APPENDIX II

A convenient example for illustrating the method of designing matching networks for series *RL* or shunt *RC* impedances is the high-frequency broadbanding of a matching transformer. Suppose a trans-

former is to be used to match a low-impedance resistive load to a high-impedance generator. The transformer is known to behave at high frequency as an inductance $L$ (leakage inductance) in series with the load resistance $R_L$. Let $\omega_h = 2R_L/L$ be the half-power angular frequency of the transformer when the load is matched to the generator at low frequencies. It is desired to broadband this transformer so as to make the loss smaller than 1 *db* up to a frequency $\omega_c = 3\omega_h/2$. Incidental dissipation will be neglected in this example.

It is convenient to normalize the network to 1 ohm impedance level, in which case the normalized inductance becomes $L_1 = L/R_L$. One has then, from Eq. 32,

$$A_1^{\infty}/\omega_c = 2/3. \tag{II-1}$$

A transmission loss of 1 *db* corresponds to a value $\ln 1/|\rho_1| = 0.79$. Figure 19 shows that the optimum tolerance for $\ln 1/|\rho_1|$ is 1.04, and that a value of 0.86 can be obtained with $n = 4$, that is, with a matching

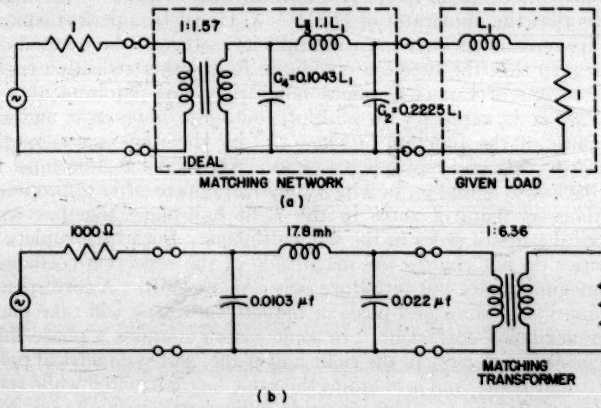



FIG. II-1. Networks for the high-frequency broadbanding of a matching transformer.

network consisting of two capacitances and one inductance. The corresponding value of $|\rho_1|_{\max}$ is 0.424 and the resulting transmission loss is 0.86 *db*. One obtains then from Fig. 18 and Eq. 36.

$$\begin{aligned} \sinh a &= 0.615, \qquad a = 0.582, \\ \sinh b &= 0.363, \qquad b = 0.356. \end{aligned} \tag{II-2}$$

The corresponding function $|\rho_1|$ is plotted in Fig. 16 *versus* the normalized frequency variable $x = \omega/\omega_c$.

In computing the element values one obtains from Eqs. 39 to 43

$$\alpha_3 = -4.493, \qquad \alpha_5 = 34.05, \qquad \alpha_7 = -435.1, \tag{II-3}$$

$$C_2 = 0.2225L_1, \qquad L_3 = 1.10L_1, \qquad C_4 = 0.1043L_1. \tag{II-4}$$

The turns ratio of the ideal transformer is specified by the zero-frequency value of $|\rho_1|$, that is, in this case, by $|\rho_1|_{\max}$. One has then

$$\text{turns ratio} = \sqrt{\frac{1 + 0.424}{1 - 0.424}} = 1.57. \qquad \text{(II-5)}$$

The resulting network for 1-ohm impedance level is shown in Fig. II-1($a$). In practice, the ideal transformer is combined with the matching transformer by performing a suitable change of impedance level. Figure II-1($b$) shows the final network for a half-power frequency $\omega_c = 50{,}000$ rad. per sec., a load impedance of 10 ohms and a source impedance of 1000 ohms.

The same design technique can be applied to the case of a load consisting of a series (or parallel) tuned circuit if the frequency band over which the load is to be matched is centered at the resonance frequency $\omega_0$ of the tuned circuit. A practical example is offered by the broadbanding of a quarter-wave grounded antenna which behaves, to a first approximation, as a series tuned circuit. Suppose, for instance, that an

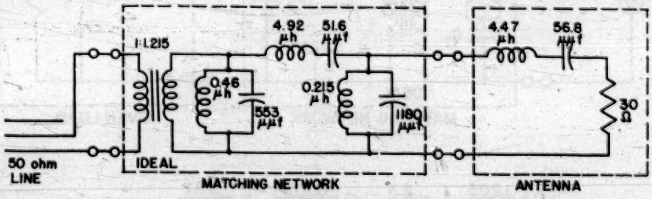



FIG. II-2. Matching network for a resonant antenna.

antenna with radiation resistance of 30 ohms, resonance frequency of 10 Mc per sec., and $Q$ of 10 is to be matched to a 50-ohm transmission line over a 3-Mc. per sec. band with a loss smaller than 1 *db*. When this band-pass problem is reduced to the equivalent low-pass problem, the same design data are obtained as in the previous example, and therefore the same basic matching network. Figure II-2 shows the network obtained by transforming the basic low-pass structure of the previous example into the appropriate band-pass structure with the required impedance level. This network can be transformed further into a chain of tuned coupled coils, and appropriate changes of impedance level can be performed so as to eliminate the ideal transformer.

## APPENDIX III

The broadbanding of the high-frequency response of a step-up transformer forms a convenient example of the technique used in matching a load impedance consisting of an inductance $L$ in series with a parallel $RC$ combination. In this case $R$ is the load resistance, $C$ is the stray capacitance of the secondary coil, and $L$ is the total leakage inductance,

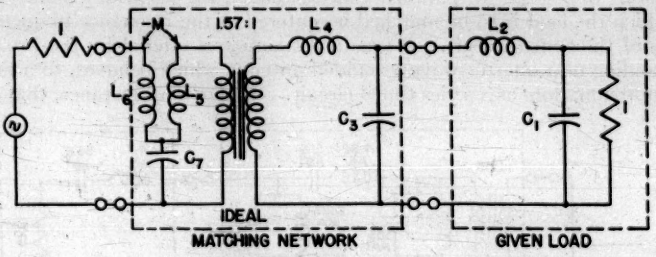
all of them referred to the primary of the transformer. With reference to Fig. III-1, suppose the normalized values of the elements forming the load impedance are given by

$$C_1 = 1.205/\omega_c, \qquad L_2 = 2.3/\omega_c,$$

where $\omega_c$ is the upper limit of the frequency band over which the impedance is to be matched. One obtains from Eqs. 44 and 45, after interchanging $C$ and $L$,

$$A_1^\infty/\omega_c = 1.66, \qquad -A_3^\infty/(A_1^\infty)^3 = 0.0475, \qquad -A_3^\infty/\omega_c^3 = 0.217.$$

The optimum tolerance for $\ln 1/|\rho_1|$ is given by the curves of Fig. 7 as 1.32, to which corresponds a value of $|\rho_1|_{\max}$ equal to 0.275.




$$C_1 = \frac{1.205}{\omega_c}\,; \quad L_2 = \frac{2.3}{\omega_c}\,; \quad C_3 = \frac{0.813}{\omega_c}\,; \quad L_4 = \frac{3.37}{\omega_c}$$

$$L_5 = \frac{0.66}{\omega_c}\,; \quad L_6 = \frac{1.135}{\omega_c}\,; \quad M = \sqrt{L_5 L_6}\,; \quad C_7 = \frac{4.63}{\omega_c}$$

FIG. III-1. High-frequency broadbanding of a matching transformer with stray capacitance loading.

To obtain the proper value of $\sinh a$, one should minimize the value of $|\rho_1|_{\max}$ of Eq. 46 subject to the conditions imposed by Eqs. 44 and 45. To avoid this tedious computation, there will be assumed for $\sinh a$ the same value that would be obtained if $L_2$ were not a part of the specified load impedance. In addition, the data of the problem have been selected, for the sake of simplicity, to yield for $n = 4$ the same values of $a$ and $b$ as in the example discussed in Appendix II, so that the maximum value of $|\rho_1|$ is still 0.424 corresponding to $\ln 1/|\rho_1|_{\max} = 0.86$. The corresponding value of $\sigma_r/\omega_c$ is 0.5.

The next step in the design is the computation of the quantities $\alpha_3$, $\alpha_5$, $\alpha_7$. In this case the quantities $2/5\sigma_r^5$ and $2/7\sigma_r^7$ must be added to the right-hand sides of Eqs. 41 and 42 to take into account the zero of

$\rho_1$ at $\sigma_r$ and the pole at $-\sigma_r$. One obtains

$$\alpha_3 = -0.523, \qquad \alpha_5 = 0.156, \qquad \alpha_7 = -0.666.$$

The values of $C_3$ and $L_4$ in Fig. III-1 are computed by means of Eqs. 40, with due regard again to the fact that $C$ and $L$ must be interchanged because the first element of the ladder is, in this case, a capacitance instead of an inductance. It results that

$$C_3 = 0.813/\omega_c, \qquad L_4 = 3.37/\omega_c.$$

The ratio of the ideal transformer is still 1.57, as in the case considered in Appendix II, but the transformer is reversed in direction because the dual network is being designed; that is, impedance has been interchanged with admittance.

In addition to the elements already computed, the presence of a zero of transmission of the matching network at $-\sigma_r$ (resulting from the zero-pole pair of $\rho_1$) leads to a Darlington section of type C illustrated in Fig. III-2. To determine the elements of this section it is

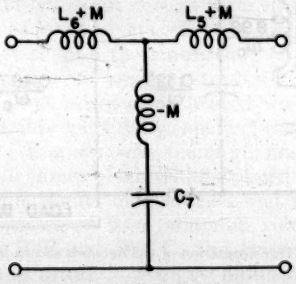



FIG. III-2. Darlington section of type C for a zero of transmission on the real axis.

convenient to operate on the reflection coefficient $\rho_2$, that is, from the opposite end of the network. It can be seen by inspection that if $M$ is a positive quantity,

$$MC_7 = 1/\sigma_r^2 = 1/4\omega_c^2.$$

At the same time, the reflection coefficient $\rho_2$ must have a zero at $\sigma_r$ and therefore the impedance measured at the $L_6$ terminals must be one for $\lambda = \sigma_r$. It follows that

$$\sigma_r L_6 + 1/(\sigma_r C_7) = 1,$$
$$\sigma_r = (1/2L_6)[1 \pm \sqrt{1 - (4L_6/C_7)}].$$

The $+$ sign must be used when $\rho_2(\lambda + \sigma_r)/(\lambda - \sigma_r)$ is positive for $\lambda = \sigma_r$. The third equation required for the determination of the three

elements is obtained by considering the quantity (see Eq. 16)

$$A_0'' = \ln \frac{\lambda - \sigma_r}{\rho_2(\lambda + \sigma_r)} - j\beta$$

which is completely determined by the elements of the section. One obtains also, from Fig. III-2,

$$A_0'' = -\tfrac{1}{2} \ln (1 - 4L_6/C_7).$$

The numerical value of $A_0''$ is found to be $-2$. Remembering that $L_5L_6 = M^2$ one has finally

$$L_6 = 1.135/\omega_c, \qquad L_5 = 0.66/\omega_c, \qquad M = 0.865/\omega_c, \qquad C_7 = 4.63/\omega_c.$$

The ideal transformer can be moved to the end of the structure and combined with the actual transformer, so that the load resistance

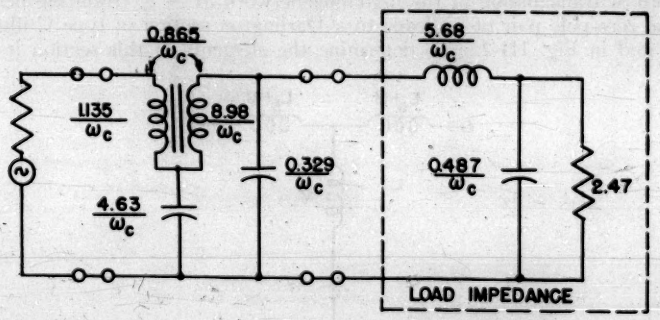



FIG. III-3. Transformation of the network shown in Fig. III-1.

measured from the primary side and normalized with respect to the source resistance will be equal to $1.57^2 = 2.47$ ohms. Finally, the coupling coefficient of the transformer in the section of type C is made smaller than unity by combining the transformer with the adjacent inductance $L_4$. The final network is shown in Fig. III-3, in which the values of all elements are normalized with respect to the source resistance.

## NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

### FATIGUE LIMIT OF CHROMIUM-PLATED STEELS FOR AIRCRAFT

Chromium plating is widely used on gages, cylinder walls, piston rings, and other machine parts where resistance to wear is an important factor. However, the advantages of chromium plating, in the absence of proper precautions, may be offset by the adverse effect of the plating on the fatigue limit of the basis metal. A recent investigation by the National Bureau of Standards provides information of interest not only to aircraft manufacturers, but also to a number of other industries which either produce or utilize chromium-plated machine parts.

Specimens about $\frac{1}{4}$ in. in diameter were machined from different types of steel, and after grinding, polishing, and chromium-plating, they were subjected to fatigue tests. Eight or ten specimens usually were required to obtain the fatigue limit for any one set of conditions. In all cases chromium plating was found to reduce the fatigue limits of the steels studied. The fatigue limit generally decreased with increased hardness of the steels and with increased temperature of the plating bath. It was not possible to make generalizations regarding the effect of plate thickness on fatigue limit though little effect was noted.

Two possible causes for the adverse effect of chromium plating on the fatigue limits of the steel were the embrittling effect of hydrogen deposited with the chromium and residual stresses in the chromium.

Chromium-plated objects are often heated with the object of improving their mechanical properties by expelling the hydrogen deposited with the chromium. A study was therefore made of the effect of heating on the fatigue limit of chromium-plated steel. The results showed that the fatigue limits of quenched and tempered specimens heated after plating decreased to a minimum value for some heating temperature between 100° and 300° C. and thereafter increased with increased heating temperatures. Fatigue limits of specimens heated for 1 hr. at 440° C. in some cases were 87.5 per cent of that of the unplated steel, whereas fatigue limits of specimens plated but not heated were only about 68 per cent.

It is generally agreed that residual stresses in a material markedly affect its fatigue limit. Thus, in steel, compressive stresses, such as are produced by shot peening, increase the fatigue limit; tensile stresses have the opposite effect. In order to show the presence of tensile stresses in electrodeposited chromium and the effect of heat treatment on these stresses, thin-walled tubes of annealed steel were chromium-plated and subsequently heated to different temperatures. After observations of heated specimens, it was concluded that the decreased fatigue limit accompanying the heating of the plated fatigue-test specimens was due to increased tensile stresses induced in the chromium plate. This conclusion has been further substantiated by experiments showing that electrodeposited chromium subjected to a heating-and-

* Communicated by the Director.

cooling cycle increases in density and thereby contracts. Since no significant dimensional changes occur in the steel upon completion of the cycle, the steel tends to restrain the complete shrinkage of the chromium layer and thereby increases the tensile stresses in it. If the chromium-plated steel is heated to 400° C. or above, the contractive forces in the chromium produce sufficient tensile stresses to cause plastic flow or rupture of the chromium plate, thus relieving the residual stresses in it and increasing the fatigue limit.

## PERMANENT GLOSS STANDARDS

Permanent gloss standards, covering the entire gloss range of nonmetallic commercial materials, are now available from the National Bureau of Standards. These standards, which may be used to calibrate any 60-degree specular glossmeter in the range from matte to high gloss, are made of ceramic materials for permanence, and the surfaces have been specially chosen for similarity to commercial materials whose gloss is important. Sets of the 60-degree specular gloss standards may be ordered from the National Bureau of Standards, Washington 25, D. C., for $70.00 a set.

Measurement of gloss is becoming increasingly important in many industries. For example, high-gloss paper is required for good reproduction of half-tone prints, and low-gloss paper for easy reading of material in type. Building interiors should range from semi-gloss to matte to prevent eyestrain caused by reflection of light sources. Outdoor finishes, such as automotive enamels, have high gloss for permanence while matte finishes are given most war goods for low visibility. Gloss measurements are also important in defining the wear resistance of plastics and ceramics.

The gloss measurement is essentially a simple one. When a specimen is illuminated at some angle from the perpendicular, the proportion of the light reflected at angles near that of mirror reflection is defined as gloss. A critical part of the definition of gloss, however, is the amount that the reflected beam may depart from the strict direction of mirror reflection and still be counted as contributing to gloss. With glossmeters commercially available at the present time, replacement of the lamp, for example, may introduce instrumental error unless readjustment is made. The new gloss standards permit a rapid check of instrument calibration at any time and give a basis for readjustment of the instrument if required.

A set of gloss standards consists of two white Vitrolite plaques having nominal gloss values 1 and 90, and eight glazed ceramic plaques with nominal gloss values 10, 20, 30, 40, 50, 60, 70, and 80, all $4\frac{1}{4}$ inches square. The plaques are calibrated with the gloss reference standards of the National Bureau of Standards. These measurements include both the true gloss and a diffuse reflectance component.

# THE FRANKLIN INSTITUTE

## MINUTES OF STATED MEETING

### January 18, 1950

The Franklin Institute held its Annual Stated Meeting in the Lecture Hall on Wednesday evening January 18, 1950, at 8:15 P.M., with approximately 200 in attendance. Mr. Nalle, the President of the Institute, was in the chair.

The minutes of the Stated Meeting for December had been printed in full in the January issue of the JOURNAL. There being no objection, they were approved as printed.

The President announced that the presentation of the annual report of the Board of Managers would be deferred until a later meeting in the spring, because the organization had grown to such large proportions that the financial statements could not be ready for presentation at this time.

The Assistant Secretary, Mr. H. V. Bail, gave a report of the new additions in the membership of the Institute for December, which were as follows:

| | |
|---|---|
| Sustaining | 6 |
| Active | 37 |
| Associate | 19 |
| Student | 13 |

The total membership as of December 31, 1949, was 5941.

The Secretary then called for the report of the tellers appointed in accordance with the by-laws to count the votes for the eight Managers to be elected at this meeting. Dr. George S. Crampton, chairman, reported the total ballots cast, and the number received by each candidate. All votes were cast for the following individuals:

E. Paul du Pont
Walton Forstall
Clarence L. Jordan
Ralph Kelly
Lionel F. Levy
Charles Penrose
James H. Robins
Philip H. Ward, Jr.

The President thereupon declared the above members elected Managers of the Institute for a term of three years. He thanked Dr. Crampton, Dr. Joseph S. Hepburn, and Mr. Penrose Hoopes for their services as tellers.

Dr. Merle M. Odgers, President of Girard College, was then introduced by the President. Dr. Odgers spoke on "Europe Revisited." He gave a delightful account of his post-war automobile tours through the war-ravaged continent of Europe and through England. His comparisons between the France of Benjamin Franklin's time and the France of 1949 amused his audience. His impressions of the reactions of the people of Europe to the Marshall Plan and America's aid in the European reconstruction efforts were particularly interesting and valuable.

H. V. BAIL
*Assistant Secretary*

## MUSEUM

The conquest of darkness by means of artificial illumination has been a problem challenging the inventive imagination ever since man invented the means of producing fire. Although the earliest known forms of lighting date from about 8000 years B.C. it is a curious fact that all those known forms are represented in the lighting methods adopted by the white man during his occupancy of America, a period which covers only a little more than four centuries. A number of exhibits illustrating the development of lighting have been added to the Chemistry Section of the Museum, and attention is drawn to the chemical aspects of the principles involved.

As long as light was produced by flame the progress was appallingly slow. Every form of oil was utilized as an illuminant but there was no appreciable change in lamp design for thousands of years. The candle is, in reality, only one form of portable oil lamp. The tallow or wax is melted by the heat of the burning wick, so that it is a liquid wax or oil which ascends the wick and assists in combustion.

The apparent stagnation becomes comprehensible when we recall that combustion was not understood until the end of the eighteenth century. With the new knowledge came the first revolutionary improvement in lamps, the Argand lamp. This was the product of a chemist and it reflected the advances in chemistry rendered possible by the discovery of the properties of oxygen.

Once science had entered the field, progress in illumination was so rapid that a method of lighting was no sooner well established than it was superseded by an improved method. Argand introduced his lamp in 1783 but before the century ended, experiments were being made with coal gas. After contemplating the abomination of using fish oil and similar odoriferous materials for the illuminant, it is amusing to read of the hysterical complaints about the introduction of the distasteful coal gas into homes. Gas succeeded in overcoming the prejudices, not only because gas jets were improved but also because chemists made great strides in the refinement of the gas itself. Indeed gas lighting had reached such an established position that it was able, wherever it was available, to fight off the encroachments of the oil lamp when it was revitalized by the introduction of kerosene.

Although coal gas figured in the next stage of development it was not in the nature of an illuminant. The Welsbach mantle attached to a gas jet gave a brilliant light, but it marked a departure from the traditional flame. Not the light but the heat from a burning gas jet was utilized to raise the temperature of the chemicals in the mantle to the point where they became incandescent. This was an entirely new approach to the problem of illumination and it inspired inventors to seek other methods of producing a brilliant incandescence, most popular of which was to obtain heat from the resistance of metals to an electric current.

The achievement of Thomas A. Edison has overshadowed that of all its predecessors, but experiments in passing an electric current through a wire in a vacuum began in England in 1809, and were continued at intervals in many countries. Of all the metals which could be made into wires or thin strips at the time, platinum and iridium were the best because of their relatively high melting points. But carbon was by no means neglected. The first American patent for an incandescent lamp was granted in 1858 to Gardiner and Blossom, who used a platinum wire.

However, Edison did much more than invent a new lamp. He developed the system by which the lamps could be used in any numbers, and he made a lamp which could be made commercially. It was essentially the Edison lamp which was improved by the introduction of new filaments and the conditions those filaments required. It is seen today side by side with still another revolutionary method of illumination. The fluorescent lamp, in which different phosphors are activated by ultra-violet radiation to produce light of different colors, is the latest form of indoor illumination.

Thus for thousands of years interior illumination underwent no change except for minor refinements, and in the relatively short period of one hundred and fifty years there have been five stages of development, each of which might be called revolutionary, since each brought into operation untried principles.

## MEMBERSHIP

### ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JANUARY 18, 1950

#### ACTIVE FAMILY

Harry Hofmeister
Donald P. Jones
Samuel L. Mumma
W. M. Scott, Jr.

#### ACTIVE

George A. Adams
Walter F. Ames
Charles O. Bateman
W. H. Beal
Henry L. Beidler
Edward H. Brautigam
John M. Broomall
Oswald S. Carliss
Edward Cohen
John A. Cornell
Charles E. Dolberg
Preston W. Foote, III
Allan A. Grant
John B. Hendrickson
Franklin Largman
George Lipschutz
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## JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Analysis.

ROOP, WENDELL P.: The Part of Octahedral Theory in the Study of the Plasticity of Metals.

LEITNER, A. AND R. D. SPENCE: The Oblate Spheroidal Wave Functions.

SILBERSTEIN, LUDWIG: Developable and Developed Silver Halide Grains.

SUSSKIND, CHARLES: Henry Cavendish, Electrician.

MICHAELSON, HERBERT B.: Variations of Grid Contact Potential and Associated Grid Currents.

FO-VAN CHANG: Trigonometric Series Applied to the Bending of Long Rectangular Plates to a Cylindrical Surface.

HERSHBERGER, W. D. AND L. E. NORTON: Servo Theory Applied to Frequency Stabilization with Spectral Lines.

HEREFORD, FRANK L.: Concerning Meson Produced Secondary Electrons Emerging from Thin Foils.

PENNER, S. S.: Application of Near-Equilibrium Criteria During Adiabatic Flow to Representative Systems.

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## THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***A Simple Television Demonstration.***[1]—ROBERT E. BENN.[2] A lecture demonstration or laboratory experiment, utilizing two cathode-ray oscilloscopes and a photomultiplier tube, provides a graphic illustration of the mode of operation of a complete television system.

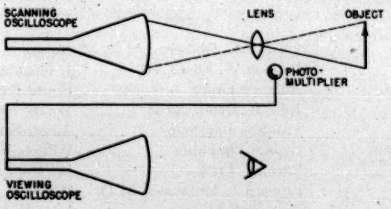


FIG. 1. Schematic diagram.

A schematic diagram of the system appears in Fig. 1. Light from a raster[3] formed by a moving spot on the screen of the first oscilloscope is focused on an object. Some of this light is reflected from the object to a photomultiplier tube connected to the second oscilloscope where an intensity-modulated pattern is produced. A reduced image of the raster (Fig. 2)

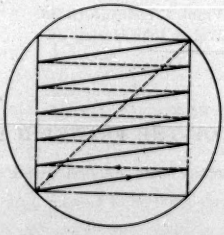

FIG. 2. Scanning and viewing raster.

appears on the object which is to be scanned. This image, while appearing to be an area of uniform illumination, is actually formed by a spot of light which traverses the image area in the same manner as in the original raster. When this spot of light falls on a region of the object having a high reflectivity, a relatively large amount of light is reflected. A portion of this light falls on the photomultiplier and a strong signal is produced. When the spot falls on a region of low reflectivity, less light is reflected and a weaker signal is produced. Thus, the signal amplitude is proportional to the reflectivity of corresponding points on the object. This signal is applied to the control grid of a cathode ray tube through the *Z*-axis amplifier, so that the intensity of the electron beam in the viewing oscilloscope is controlled point by point in accordance with the reflectivity of the scanned object. Light shields may be provided to prevent extraneous light from falling on the phototube.

Similar rasters are displayed on the screens of the two oscilloscopes. Each raster resembles that shown in Fig. 2, where the arrows indicate the direction of movement of the spot of light.

[1] Reprinted from *American Journal of Physics*, Vol. 17, No. 7, 437–438, October, 1949.
[2] Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.
[3] Raster—a television term, defined as a predetermined pattern of scanning lines which provides substantially uniform coverage of an area.

A high frequency sawtooth voltage applied to one pair of deflection plates in each cathode ray tube causes the beam to move across the screen at a constant velocity, as shown by the solid lines, and return with a high velocity, as shown by the dashed lines. Simultaneously, a low frequency sawtooth voltage applied to the other deflection plates causes the spot to move in a perpendicualr direction at constant velocity. The displacement due to this latter movement separates the lines and thereby forms the raster. At the end of its travel the spot returns with high velocity to the starting point and begins tracing out the next cycle. By varying the frequency ratio of the two sweep voltages, the number of lines per frame can be varied.

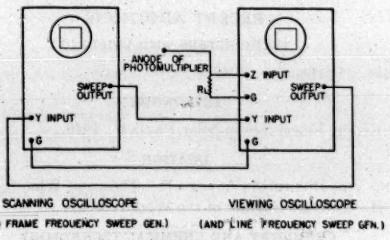


FIG. 3. Diagram of connections.

A diagram of connections appears in Fig. 3. The rasters are generated by operating the internal sweep generator of the viewing oscilloscope at the line frequency and the internal sweep generator of the scanning oscilloscope at the frame frequency. Since the sweep voltage of each is applied to the *Y*-axis amplifier of the other, similar rasters appear on the two oscilloscopes. It should be noted that this method causes the scanning raster to be rotated 90° relative to the viewing raster. If the viewed image is to appear in its normal position, the scanned object should be rotated through a right angle.

Although almost any cathode ray oscilloscope can be used, with slight modifications, the Dumont Type 247 is ideal because the internally generated sweep-voltage is available externally at a terminal, and because it contains a suitable *Z*-axis amplifier. The photomultiplier used is a 931-A, and the cathode ray tubes are Type 5CP11-A. The phosphor color on the viewing tube is immaterial but that of the scanning tube should be P11 since its peak emission falls near the peak sensitivity of the 931-A. The multiplier voltage, 112½ volts per stage, is secured from a bank of small 45-v dry batteries with 22½-v taps.

The bandwidth of the system is limited primarily by the decay time of the light from the scanning tube phosphor. Since this decay is not instantaneous, the scanning spot tails out and limits the obtainable resolution. The multiplier load resistor, $R_L$, can be as high as 50,000 ohms without materially reducing the resolution.

The frame frequency should be high enough to eliminate flicker, about 30 cycle $sec^{-1}$. The line frequency can be varied to control the number of lines in the picture, but the limit imposed by considerations of resolution is about 3000 cycle $sec^{-1}$. (100 lines).

Using an *f* 3.5 lens of 50-mm focal length with the scanner, this system can transmit printed matter, line drawings, and simple pictures with considerable clarity. The scanned area must be kept small, between ½ inch and 1 inch on a side, in order that the amount of detail in the picture shall be within the capabilities of the system, and the intensity of the reflected light high enough to provide a sufficiently strong video signal. This equipment presumably could be used to reproduce photographic negatives by equipping the multiplier with a lens and placing the negative between multiplier and scanner. Various experiments on resolution and band width that can easily be performed demand only minor additions to the equipment.

*Engravings through the courtesy of the American Journal of Physics*

## LIBRARY

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*Abstract of* **A Polysaccharide Related to the Blood Group Substances and its Reaction with Borate. I. A Study by Electrophoresis.**—LAURA E. KREJCI, LUCILE SWEENY, AND CHARLES A. ZITTLE (*Arch. Biochem.*, **22**:253, 1949). A polysaccharide isolated from calf intestinal mucosa, and related serologically to the blood group A and B substances, exhibits increased mobility in the presence of borate ions through formation of diol-borate compounds of increased acidity. The mobility increments were found to vary with (1) pH, (2) borate ion concentration, and (3) polysaccharide concentration. Together, (2) and (3) determine the concentrations of the diol-borate compounds, and (1) determines the degree of ionization of those compounds. A sharpening of the electrophoresis boundaries paralleled the rise in mobility.

*Abstract of* **A Polysaccharide Related to the Blood Group Substances and its Reaction with Borate. II. A Study by Sedimentation and Viscosity.**—LAURA E. KREJCI, LUCILE SWEENY, AND JOHN HAMBLETON (*Arch. Biochem.*, **22**:262, 1949). At pH 8.6, sodium borate, in concentrations up to 0.05 *M*, progressively reduced the viscosity of dilute solutions of blood group polysaccharide and increased the sedimentation constant gradually from 7.1 to 8.0 *s*. In concentrations up to 0.025 *M*, sodium borate increased the viscosity of concentrated solutions and produced a further increase in the sedimentation constant; in 0.05 *M* sodium borate, on the other hand, concentrated solutions as well as dilute solutions were reduced in viscosity, and the sedimentation constant was independent of polysaccharide concentration throughout the the same range as in borate-free solution. These changes may be attributed to predominance of mono-diol borate combination at low polysaccharide concentrations, or at high polysaccharide concentrations when the ratio of polysaccharide to borate is low, and the occurrence of di-diol borate combination, which links adjacent polysaccharide chains, at high concentrations of polysaccharide when the ratio of polysaccharide to borate is high.

*Abstract of* **The Occurrence of Vegetative Cells of *Clostridium Perfringens* in Soil.**—LOUIS DESPAIN SMITH AND MARY VIRGINIA GARDNER (*J. Bact.*, **58**:407, 1949). In order to obtain information on the form in which *C. perfringens* exists in the soil, the relative numbers of heat-resistant and heat-susceptible forms were determined in eight samples of soil which had not been exposed to known fecal contamination for some years. It was found that large numbers of *C. perfringens* can exist in the soil in a form susceptible to 78° C. for 20 min. Since these susceptible forms are probably vegetative cells, multiplication of *C. perfringens* must take place in the soil and soil must be a natural habitat of this species.

## BOOK REVIEWS

APPLIED EXPERIMENTAL PSYCHOLOGY, by Alphonse Chapanis, Wendell R. Garner, and Clifford T. Morgan. 434 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $4.50.

The authors of this new book have tried an experiment, the success of which cannot be assessed at this time, in writing in a novel style upon a subject of mutual interest to systems equipment design engineers and psychologists. Applied experimental psychology can be defined as the application of the knowledge of the capabilities of human beings to the design of instruments and combinations of instruments to form systems. The authors, all practicing psychologists of high repute, have presented in simple terms in this working encyclopedia for engineers an account of the many factors which certain engineers should recognize. Modern technology has been advancing rapidly, but there has been only a slow application of the findings of the so-called experimental engineering psychologists. The fundamental knowledge of human capacities, human performance, and the operating combinations of humans and machines, subjects of prime interest to some psychologists, can be effectively applied to the design and operation of machines for human use. The authors indicate how this can be done.

The topical organization of the book makes for continued interest. Two chapters, devoted to elementary statistics useful in performing tests and appraising and evaluating equipment serve as introduction. There then follow chapters devoted to a presentation of the basic facts pertaining to human sensations and perceptions—how man sees, hears, and moves. Information of this kind can be considered as input data to the human as an engineering operator. The human being, operating upon the input signals, performs certain functions which can be considered as outputs of the human as a machine. With this scheme in mind, a series of three chapters considers: the fundamentals of sight, visual acuity, and optical illusions; instrument dials and legibility; and visual displays.

A similarly organized triad presents: the fundamentals of hearing and sound; speech intelligibility; and tone signalling systems. Movements and reactions, controls for human use, and arrangements of working areas form another set of three chapters. The last two chapters are devoted to fatigue problems and the working environment.

Much has to be said about the book in addition to the description of the contents. The authors have made a rather thorough search through the literature, as represented by the many references listed, and have in the case of each major topic, presented tables, sketches, and graphs illustrating in concise form the findings of past work. The vast amount of practical data presented in this manner can be applied with almost no change by design engineers, if the design engineers would only have confidence in the findings of the psychologist. This book, when appreciated by the engineer, may help destroy the barrier which now exists between many engineers and psychologists who do not appreciate the mutual nature of problems dependent for solution upon a harmonious working together of men and machines.

This reviewer cannot close without two remarks which may be considered in any future editions: (1) During the past war much basic study on the subject of tracking was conducted at The Franklin Institute, and no reference is made to any of this work; (2) In the preface the authors somewhat deprecate engineers by the statement "We were academics talking about technical psychological matters to an engineering audience. For this reason we had made deliberate efforts to use small words instead of big ones—." While such statements were not designed to ruffle the feathers of the engineering fraternity, it is suggested that future editions of the book should modify such statements.

S. CHARP

ELEMENTS OF THERMODYNAMICS AND HEAT TRANSFER, by Edward F. Obert. 372 pages, illustrations, plates, 16 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $4.50.

This book has been written by the author for use as a beginning text in undergraduate engineering courses in thermodynamics and heat transfer. It has been organized for one- or two-semester courses, and is essentially a simplified version of the author's more comprehensive "Thermodynamics" published in 1948.

The presentation is straightforward and logical. Engineering thermodynamic problems encountered in real machines and flow processes have been emphasized. The author has made a careful effort to introduce the fundamental concepts of thermodynamics to the engineering student as simply and directly as possible, without the usual large number of equations at the beginning of the book, but without sacrificing any of the rigor or continuity of the science. The book is amply illustrated by diagrams and graphs of thermodynamic systems, drawings and photographs of thermodynamic machines, and by charts and tables of properties of gases, vapors, and fluids.

The text begins with a survey of engineering dimensions and units, and the fundamental thermodynamic concepts of energy and systems. The First Law as expressed by the general energy equation is introduced, accompanied by numerous practical applications to boilers, reciprocating and rotary pumps, nozzles, turbines, and condensers. The concept of reversibility is presented next, and is expanded by an explanation of work in closed and steady-flow open systems. Finally, the Second Law is defined, along with the thermodynamic concept of entropy, and the Carnot cycle.

Upon this framework of fundamental concepts the properties of real fluids, the characteristics of perfect and real gases, the equations of flow of fluids, the characteristics of mixtures of gases and vapors, and the thermochemical calculations of combustion are presented.

Applications of thermodynamic concepts and equations to practical systems is introduced by a presentation of vapor power cycles including the Carnot and Rankine cycles. This application is expanded to gas power cycles including the Otto, Diesel, and Brayton (Joule) cycles. The refrigeration cycle and the concept of heat pumps are finally presented.

The last chapter is a fundamental survey of the principals and equations of heat transfer by conduction, convection, and radiation.

"Elements of Thermodynamics and Heat Transfer" is a well written and illustrated, fundamental text in thermodynamics for engineering students which should find general acceptance in colleges and universities throughout the United States.

RICHARD A. WALLACE

ACETYLENE AND CARBON MONOXIDE CHEMISTRY, by John W. Copenhaver and Maurice H. Bigelow. 357 pages, illustrations, 15 × 24 cm. New York, Reinhold Publishing Co., 1949. Price, $10.00.

As early as 1925 German industrialists were interested in developing a source of synthetic rubber from the cheap and readily available compound, acetylene. The initial task was to find an economically feasible method of synthesizing butadiene, a starting material of synthetic rubber, from acetylene. This project was given to one of Germany's leading chemists, Dr. Julius Walter Reppe.

Reppe successfully completed his butadiene project by 1930, then he and his group turned their interests to new syntheses involving the very reactive acetylene. By carefully controlling experiment conditions and techniques and by working at pressures of twenty to thirty atmospheres, an entirely new source of organic intermediates became apparent. Dr. Reppe and his associates began to apply these new found techniques to three of the most important reactions of acetylene; namely, vinylation, ethinylation, and polymerization. This work provided a large and relatively cheap source of compounds useful in the plastics industries.

A by-product of the manufacture of acetylene is carbon monoxide, and hence the logical step would be to attempt to cause it to react with acetylene thus utilizing both product and by-product of the acetylene synthesis. After much tedious work, Reppe found that the heavy

metal carbonyls were the best catalysts for this, the carboxylation reaction, from which acrylic esters as well as several hydroquinones can be made.

Germany, in its plan to become industrially independent and to wage war, was drastically in need of sources of plastics to supplant natural materials such as rubber, resins, and various metals which had to be imported. Factories were built, therefore, to fully utilize the techniques developed by Reppe, and many of them were in production during the recent war. The vast economic and academic importance of this research was apparent to the Allied Governments, and the need for an accurate and composite record of the work was essential. The gathering, translating, and rewriting of almost thirty tons of documents scattered all over war-torn Germany was given to Bigelow and Copenhaver. Together with Reppe, who was completely reluctant to help these enemies of his country, the work was gradually accomplished and, even though by no means complete it stands unique in the field.

The book may be divided into five parts; 1. Synthesis and Reaction of Acetylene under Pressure, 2. Polymerization of Acetylene, 3. The Carboxylation Reaction, 4. Synthesis and Reaction of Homologs of Acetylene, and 5. Techniques for the Safe Handling of Acetylene under Pressure. The authors successfully trace the development of this new branch of chemistry and include, besides the reactions themselves, thorough engineering details of equipment, catalysts, yields, and even many of Reppe's own theories. A general bibliography of German acetylene chemistry is included to provide a source of detailed information for those interested.

All of the basic work is covered in the book, but many of the reactions are given rather cursory treatment. However, being confined to a single volume, some material had to be deleted. It is written in a straightforward and easily readable style, and a flow sheet of reactions is put at the end of each chapter to facilitate the remembering of the huge amount of subject matter. The book is offered as a sort of handbook of information in this new field, and even though it is not complete its merits far outweigh this shortcoming. The book should be a valuable addition to the library of the chemist and research worker in the field of acetylene chemistry.

DONALD H. RUSSELL

PRINCIPLES OF AERODYMANICS, by James H. Dwinnell. 391 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $5.50.

This is a distinctive text written specifically as an undergraduate course on the theory and application of aerodynamics. Its theoretical and practical aspects are ably presented by the author, and a wealth of experimental data is included for comparison with theory. The theme of the book may best be described as the unfolding of aerodynamic principles and the verification of these principles by recent test data.

The first few chapters deal with the fundamentals of motion, the air properties, aeronautical nomenclature, and experimental facilities. This is an absorbing book for the student and familiarizes him with fluid dynamics, airfoil characteristics, the influences of aspect ratio and wing plan-form, viscosity phenomena and compressibility phenomena which form a good foundation for any advanced studies in the field of aerodynamics.

A chapter is devoted to the development of the airfoil and includes many test data on the present day low-drag type. The use of auxiliary lift devices, boundary-layer control, and dive brakes is discussed, using illustrations, thus pictorially showing their effect.

Airplane performance, stability and control are discussed in detail which familiarizes the student not only with the usual means for their determination, but discusses also parasite-drag and power considerations, aircraft engines, and propellers.

Throughout the book there are many numerical examples used to clarify explanations and theoretical concepts. A wealth of illustrations is employed and the photographs of experimental testings are of excellent reproduction.

Although written principally for the student, this book is a valuable asset for aerodynamical reference.

MAX W. BERG

RAILROADS OF TODAY, by S. Kip Farrington. 306 pages, plates, 15 × 22 cm. New York, Coward-McCann, Inc., 1949. Price, $6.00.

For the railroad fan the appearance of a new railroad book is always a welcome event. But this book should also have a wide appeal to the non-fan, for even though he never travels a train, he benefits indirectly from railroad service, and here Mr. Farrington presents a picture of railroads today and of their efforts to make their service better. The author of several railroad books, he describes the details of many of the modern developments in this vital field of transportation, using different railroads as typical examples.

After an introductory chapter in which the author takes the reader for a ride on typical but mythical coach and Pullman trains, he proceeds to a description of the development of the modern Diesel, which on some roads has completely replaced the steam locomotive. Then is introduced the new Twentieth Century Limited, still a name to conjure with in this age of many great name trains. The equipment is described in considerable detail and adequately conveys the many comforts and conveniences to be found on this train. Throughout the book there are descriptions of other fine trains such as the Powhatan Arrow of the Norfolk and Western, the Empire Builder of the Great Northern, and the Cincinnatian of the Baltimore and Ohio. Other chapters are devoted to such specialized types of trains as mail trains on the Santa Fe, fruit trains on the Missouri Pacific and stock trains on the Union Pacific.

Foremost among recent technical developments has been centralized traffic control or C.T.C. After discussion of coded track circuits and their use in signalling, Mr. Farrington gives an account of the subsequent development of C.T.C. which permits a train dispatcher to handle traffic one hundred miles or more away from his office. This one improvement has enormously increased the capacity of the railroads to handle traffic, especially when it has been installed on single track lines. Examples of C.T.C. in action are given on such varied roads as the Norfolk and Western, Santa Fe and Union Pacific.

Other technical developments and improvements noted are the Chesapeake and Ohio's "500" steam turbine locomotive, roller bearings on cars, new motor car signals, and retarder switching yards installed on the Union Pacific at Pocatello and North Platte. The railroads have also introduced new service features such as the "Sentinel Service" of the Baltimore and Ohio for speeding freight shipments, and the central ticket reservation system of the Chesapeake and Ohio.

As in others of Mr. Farrington's books, there is a large number of photographs of railroads in action which will delight any railroad fan. The shots of locomotives are about equally divided between steam and Diesel. One minor criticism of the book that might be offered is that Mr. Farrington has been rather repetitious of some of his facts, when there seems nothing to be gained by such repetition.

This volume is one that will entertain and inform those who are already railroad enthusiasts, and for those who are not, it will serve as an excellent account of the efforts the railroads are making to offer improved service to the public. That so much has been accomplished in the face of subsidized bus and air competition and heavily increased costs, is a tribute to the vitality and generally good management of the railroads.

G. E. PETTENGILL

## BOOK NOTES

INDUSTRIAL CHEMISTRY, AN ELEMENTARY TREATISE FOR THE STUDENT AND GENERAL READER, by Emil R. Riegel. Fifth edition, 1015 pages, illustrations, diagrams, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $7.00.

This well-known textbook has been largely rewritten and brought up to date. Among the chapters which have undergone extensive revision are those on glass, sugar, water and sewage, petroleum, photography and the utilization of wood. New processes have been described and new products and uses included. As in the earlier editions, the author has had the collaboration and advice of numerous specialists.

OUTLINES OF FOOD TECHNOLOGY, by Harry W. von Loesecke. Second edition, 585 pages, illustrations, diagrams, tables, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $7.50.

Since the appearance of the first edition of this work some eight years ago, there have been significant improvements in food processing. This new edition has incorporated not only a discussion of these new techniques, but also a consideration of new products and advances in design of equipment. Advantage has also been taken of the opportunity to eliminate the errors of the earlier edition.

The general scope of the work remains the same, to describe the more important food processes and their application to different foods. Ample references to the literature are included for those desiring further information.

THE AMERICAN ANNUAL OF PHOTOGRAPHY, 1950. Volume 64, edited by Frank R. Fraprie and Franklin I. Jordan. 246 pages, illustrations, plates, 18 × 25 cm. St. Paul, American Photographic Publishing Co., 1949. Price: paper, $2.00; cloth, $3.00.

This edition of the *American Annual of Photography* should appeal to more than just the photographically inclined readers of the JOURNAL, for it contains several articles bearing on scientific and industrial applications of photography. The longest is entitled "Photography in Industry and Science" by Glenn E. Matthews, in which he points out four broad types of uses. These are: 1. simple records of objects and events; 2. multiple records for observing and measuring changes which occur with time; 3. records of fast-moving and of slow-moving events; and 4. records of invisible phenomena. Various examples of each of these types are given with illustrations, among which might be mentioned newer techniques in aerial photography, ultrafax, and radiographic applications.

Henry M. Lester in his article "The Motion-Picture Camera in Science and Industry" emphasizes the use of the camera as a *research instrument*. Ronald L. Ives offers some comments on the selection of photographic equipment for use in the field sciences—geology, geography, meteorology, and others, in his paper "The Camera as a Field Research Tool."

Technical material on photography itself is offered in articles on developers and development, printing exposure determination by photoelectric methods, contrast control processes, and brown toning.

The artistic side of photography is treated in articles discussing children, nature, snow scenes and airports as subjects. The work of José Ortiz-Echagüe is described and depicted at some length. The customary section of pictorial illustrations is represented by an interesting selection of prints.

A variety of subject matter insures a wide appeal for this volume.

GRUNDZUEGE DER ANGEWANDTEN GEOELEKTRIK, by Volker Fritsch. 412 pages, illustrations, diagrams, tables, 15 × 23 cm. Wien, Manzsche Verlags-und Universitätsbuchhandlung, 1949. Price not given.

In this volume Dr. Fritsch treats of that phase of geophysics which considers the electrical characteristics of the subsoil and the bodies contained therein. Because of the many practical applications which can be made of geoelectrical techniques, the subject is one which merits this study giving an over-all view.

In his treatment the author considers first the geological conductors and presents tables of conductivity, dielectric constant and resistance. Then he developes general thoughts on geoelectrical measurements before proceeding to a consideration of the various techniques. These include both direct and alternating current methods, the latter in low and middle frequency as well as in high frequency. Following the description of techniques Dr. Fritsch offers a discussion of the evaluation of measurements and presents the physical hypotheses for the interpretation of the resistance measurements. The concluding chapter considers the various practical applications of geoelectrical techniques, among which may be mentioned the investigation of metallic deposits, coal and petroleum, the proof of water sources, the proof of quicksand, and building ground investigations.

The work is detailed and includes many diagrams of circuits and instruments. It is supplemented by a lengthy bibliography of some fifty pages, giving a classified approach to the literature.

ANNUAL REPORTS ON THE PROGRESS OF CHEMISTRY FOR 1948. 379 pages, 14 × 21 cm. London, Chemical Society, 1949. Price, £ 1.5.0 d.

This is the forty-fifth volume to appear in this important review series. As in other years the articles consist primarily of reviews of specific topics, although for inorganic chemistry there is a review of the whole field. A brief statement of the contents will probably best suffice for the purposes of this note to indicate the topics included. In the field of general and physical chemistry, the reviews cover chemical reactions induced by ionizing radiations; structure of aqueous solutions of soap-like substances, and chemical kinetics: homogeneous thermal gas reactions. The organic chemistry division treats of general methods, homolytic reactions, terpenes, colchicine and related compounds, organic sulfur compounds, and heterocyclic compounds. In biochemistry, there are articles on the function of small molecules in biosynthesis, partition chromatography, the chemistry of insulin and the chemotherapeutic approaches to the T.B. problem. The analysis of sea water is one of the subjects considered under the broad head of analytical chemistry, the others being analytical emission spectrography, flame photometry and volumetric analysis.

THE BUSINESS HELPER, by Leslie C. Rucker. 133 pages, 13 × 19 cm. New York, John F. Rider Publisher, Inc., 1949. Price, $2.00.

The author has compiled a collection of hints on running a small business profitably, which should prove helpful for the business man with little training or experience. No attempt has been made to be technical; in fact the book is quite elementary both in concept and presentation. It covers such topics as locations, buying, selling, overhead expenses, advertising and insurance. As a reflection of twenty-three years of experience, the ideas presented may be considered workable.

BORON TRIFLUORIDE AND ITS DERIVATIVES, by Harold S. Booth and Donald R. Martin. 315 pages, illustrations, diagrams, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

In 1936 commercial production of boron trifluoride was begun by the Harshaw Chemical Company and in the subsequent twelve years there was a great increase of knowledge in this field. The present monograph aims to give an organized presentation of the available facts about boron trifluoride, as of January 1, 1948. The authors have a good background for their task, as the senior author has been investigating fluoride gases since 1915 and much of his work has dealt with the chemistry of boron trifluoride and the junior author has studied the boron halides with emphasis on accounting for the mode of formation of coordination compounds.

After a brief summary of the history and preparation of boron fluoride, the authors devote three chapters to the physical properties, chemical properties and the coordinating power of boron trifluoride. The various fluoboric acids and their salts are considered in detail. As catalysts, boron trifluoride and its derivatives have assumed considerable importance and this phase of the subject has been treated at length. Analysis and comments on the practical handling of the compound have also been included.

An exhaustive bibliography with well over a thousand entries has been compiled. In listing patents the authors have had the cooperation of eighteen companies in the field, while throughout the work they have had advice and assistance from the Harshaw Chemical Company, to whom they make a grateful acknowledgment. This volume should prove valuable as a summary of the available knowledge on boron trifluoride and as a basis for further research.

## CURRENT TOPICS

**New Process for Stabilizing Guncotton Aids National Defense.**—A new method of stabilizing guncotton, which saves about two-thirds of the time formerly needed for making this important explosive, has been developed by the Southern Regional Research Laboratory of the Bureau of Agricultural and Industrial Chemistry, it is announced by the U. S. Department of Agriculture. Because it makes possible a substantial cut in the cost of smokeless powder for large-caliber guns, the laboratory's process can help taxpayers get more for their defense dollars. The new method also permits powder factories to be smaller and thus less vulnerable to air attack in the event of war, as well as easier to move underground if necessary.

Guncotton, or nitrocellulose, is the main ingredient of smokeless powder, the propellent explosive used in firing shells from big artillery rifles and naval guns. It is made by treating cellulose—usually cotton linters, the short fibers left on cottonseed after ordinary ginning—with a mixture of nitric and sulfuric acids. The nitric acid combines chemically with the cellulose, transforming it into guncotton. The sulfuric acid, which acts as a sort of catalyst in the process, must later be washed and boiled out. Even small amounts of this acid left in the finished nitrocellulose make it unstable, or likely to decompose. This instability may result in a gradual loss of the guncotton's explosive power or a change in its rate of power release. Under certain conditions, incompletely purified nitrocellulose may explode prematurely. Removing all traces of the sulfuric acid is difficult, and the conventional procedure for purifying and stabilizing the explosive is long, laborious, and expensive.

The new procedure, devised by Richard E. Reeves and coworkers at the Bureau's New Orleans laboratory, takes much less time and requires much smaller operating space and less fuel than the conventional process. It removes most of the sulfuric acid by washing and boiling, as in the usual method, but in one-third to one-fourth the usual time—that is, in 20 hours instead of the customary 60 to 80 hours. All remaining traces of acid are then neutralized quickly with ammonia, a procedure which makes further boiling of the guncotton unnecessary. This final step of the new process may be carried out either at ordinary or at elevated temperatures.

Development of the new stabilizing method was a result of research undertaken by the laboratory during the war to find out whether staple cotton cut to short lengths could be used in smokeless powder to supplement the limited U. S. supply of cotton linters. Laboratory chemists nitrated the cut cotton fibers to see if they were suitable as guncotton (which they were) and incidentally hit upon the new method of stabilizing nitrocellulose. Their findings were immediately made available to the armed forces.

Smokeless powder made with nitrocellulose stabilized by the ammonia procedure has passed all preliminary tests and is now undergoing long-term storage trials. Investigation of the powder's stability over long periods is essential to make certain that nitrocellulose prepared by this method will perform, when needed, exactly in accordance with previous ballistic determinations.

**Continuous, R-F Induction Selective Hardening Equipment.**—Equipment for the continuous, r-f selective induction hardening of cylindrical parts at feed rates to six inches per second, is available from Westinghouse Electric Corporation.

This equipment consists of three major components; an Automatic Loading Device, a Horizontal Rotating Scanner, and an Industrial Radio-Frequency Generator.

The Automatic Loader, consisting of a magazine and hopper, feeds the pieces to be hardened to the scanner through a feeder unit. The equipment hardens by passing the work through an inductor coil and spray quench ring. Uniformity of case depth is obtained by controlled feeds; concentricity of case is assured by positive guide and rotation of the cylindrical work piece about its longitudinal axis.

This equipment can be used to harden a wide variety of cylindrical parts in any desired hardness pattern by simple adjustment of electronic timing circuits.

**Clay Improvement Process.**—Thanks to two University of Wisconsin professors, brick and tile manufacturers today can use a simple process which will both cut production costs and production headaches and will also make sounder clay products.

The production method is the Barker-Truog Clay Improvement process. Simply put, it involves the *controlled* addition of soda ash to clay. It is the result of years of painstaking laboratory experiments at Madison by George J. Barker, professor of mining and metallurgy, and Emil Truog, professor of soils.

The process has been patented by the scientists. They in turn have assigned all patent rights to the Wisconsin Alumni Research foundation of Madison. The foundation licenses use of the process in exchange for minute royalties.

Professors Barker and Truog discovered their clay improvement process just prior to World War II. Several brick and tile plants put it into use immediately, with highly satisfactory results.

But before the B-T process could spread throughout the structural clay industry, soda ash became a critical defense item. Only in the past year has soda ash again become available. Now the B-T process can be employed cheaply and easily throughout the country to produce better clay products for less money.

Because of their unusual characteristics, some clays and shales may not respond sufficiently to make the use of the B-T process feasible, but ordinarily the B-T process will mean choicer, tougher, more durable structural clay products, according to Boynton Butler, general manager of the Research foundation.

To use the B-T process, changes in plant procedure or equipment are not necessary. In most cases, all that is needed is an inexpensive device for feeding a controlled amount of soda ash.

**A New Science Service for Professionals and Laymen.**—Of great interest to professionals and students in every field of science is the new time-saving

service recently formed by the Science Book and Information Service. This organization, geared to the needs so frequently felt by scholars and researchers in an era of intense specialization, offers a many-sided service:

(1) A worker specializing in any field may use its personal book-information section designed to meet his individual requirements. (2) Those who have been seeking rare or out-of-print books may secure them through the Service. (3) Anyone who wants to be kept continuously up to date on developments, whether in a specific subject or in general science, may readily do so through the regular bulletins, issued monthly. These bulletins, available separately in each subject, supply comprehensive information (summaries, descriptions, listings), on every scientific and technical book of importance in the field, whether already published or forthcoming. The SBIS also services its members by supplying desired books upon request.

Readers and practicing scientists thus have at hand complete authoritative information on any subject, pooled into one easily referable source. A similar service is available to technicians, craftsmen, hobbyists, and readers with a lay interest in science.

Readers wishing further information may have it promptly by writing (stating special field of interest) to Science Book and Information Service, 471 Orange Street, S.E., Washington 20, D. C.

**Baltimore Installs Harbor Radar.**—Baltimore recently became the third major port in the world to put into operation a harbor radar system.

The equipment will be used in a navigational aid research program designed to assist ships entering and leaving the port in fog and bad weather, to provide continuous observation of harbor shipping, and to give immediate information on the location of any shipping casualties in the harbor.

The radar equipment, a Westinghouse commercial marine radar unit, provides port operators with a 12½ in. radar "chart" of harbor shipping movements at ranges from 80 yards to 40 miles. It is installed at the City Recreation Pier in the radio control room, so that radar observations can be transmitted directly to harbor shipping over stations WMH and WJY, the city's ship-to-shore radio stations.

The radar, manufactured at the Wilkens Avenue plant of the Westinghouse Electronics and X-Ray Division, has been made available to the city on a long-term loan by the Division. The only other ports in the world equipped with radar are Long Beach, California, and Liverpool, England.

Mr. Tull explained that the radar equipment had been installed at the Recreation Pier because this position permitted a clear view of the active inner harbor, allowed direct communication to vessels, and was convenient to the Chesapeake Bay Pilots Association offices which are in the same building.

**Accurate photo-copies of line drawings to scale** are greatly speeded through use of a method suggested by a graphic-arts mechanic at the Norfolk Naval Air Station. Reversing the normal method, the rear of the copy camera is made movable, with the lens fixed. Graduated scales are installed on the bed for use in measuring the size of copy desired, thus making it possible to turn out copies quickly and without unnecessary wastage of film due to guesswork.

# BY-LAWS

## ARTICLE I

### Stock

*Section 1.* The Real and Personal Estates of the Institute as held upon the First day of January, One Thousand Eight Hundred and Eighty-one, shall be valued at One Hundred Thousand Dollars, and shall be represented by Ten Thousand Shares of Stock of the par value of Ten Dollars each. Said shares shall be divided into two classes, *viz.:*

First Class. Shares not registered for use: on which no annual payment shall be charged or collected, and the holders thereof shall not have the privileges of members of the Institute, but may, if of legal age, vote at any annual election for managers upon the payment of One Dollar upon each share of stock on which they may desire to vote; provided, however, such shares have been held by the same person at least three months before such election.

Shares of the First Class may be converted into shares of the Second Class at the pleasure of the owners, provided the transfer be approved by the Board of Managers; but, when once so converted, they shall always continue in the Second Class.

Second Class. Shares registered for use: on which Twelve Dollars per annum shall be due and payable from resident members in advance on the first day of October in each year, except as hereinafter provided.

Non-resident holders of Second Class stock shall pay an annual fee of Five Dollars.

*Section 2.* The holders of Second Class stock shall be entitled to the use of the library, lectures and reading-room, and if of legal age, to all other privileges of membership in the Institute, so long as they make the annual payment in advance; and shall, on the payment of One Dollar therefore, be entitled to a Certificate of Membership.

*Section 3.* If the annual dues for successive years remain unpaid at the expiration of two and a half years on any share of stock of the Second Class, such shares shall then become forfeited to the Institute; but such forfeiture may be remitted by a unanimous vote of the Board of Managers.

*Section 4.* Stock of the Second Class may be held in trust for persons not of legal age, and shall be liable to the payment of only one-half the annual fees due upon stock of Second Class held by persons of legal age; provided, that when such minors arrive at legal age, new certificates, subject to the full annual contribution, shall issue on payment of the customary fee.

*Section 5.* Certificates for the First Class stock may be issued for any number of shares in a single certificate; but every certificate for the Second Class shall be for one share only.

*Section 6.* No share of stock in the Second Class shall be transferred until all arrearages and fines are paid, and all books and tickets returned, and the transfer approved by the Board of Managers.

*Section 7.* All certificates of stock shall be signed by the President and Secretary; shall be issued by the Controller, and shall be transferabe only on the books of the Institute by the owner, or his legal representative, on the surrender of the old certificate, and of a fee of twenty-five cents for each certificate.

*Section 8.* All subscriptions to stock shall be approved by the Board of Managers before the certificate can be issued.

## ARTICLE II

### Members

*Section 1.* All persons interested in the purposes and activities of the Institute and willing to further them may become members when elected by the Board of Managers, or in a manner prescribed by the Board, except as qualified by Section 6 of this Article. Membership shall consist of the following classes:

1. Student members
2. Associate members
3. Active members
4. Sustaining members
5. Honorary members

The Board of Managers may establish other classes of members, provided that the privileges enjoyed and dues paid by such other classes are not inconsistent with the privileges and dues of the classes of members specifically provided for by these By-Laws.

*Section 2.* Student members shall be under twenty-five years of age and shall pay annual

dues of $2.00. They shall be entitled to unlimited free admission to the Museum and the Planetarium. Student members who are fourteen or more years of age may, if endorsed by a teacher or an Active member, have the use of the library upon the payment of additional annual dues of $1.00 but shall have no voting privileges or rights to hold office.

*Section 3.* Associate members shall pay annual dues of $5.00. They shall be entitled to unlimited free admission to the Museum and to the Planetarium, but shall have no voting privileges or rights to hold office. Upon the payment of $5.00 additional annual dues they shall have Family privileges as defined in Section 7 of this ARTICLE.

*Section 4.* Active members shall be not less than twenty years of age. They shall pay annual dues of $15.00. Active members, residing permanently at a distance of fifty miles or more from Philadelphia, shall pay annual dues of $7.50. Active members shall be entitled to use the library, to receive one copy of the Journal of The Franklin Institute, to vote and to hold office. They shall be entitled to all the privileges of Associate members and upon the payment of $5.00 additional annual dues shall be entitled to Family privileges.

*Section 5.* Sustaining members shall be not less than twenty years of age. They shall pay annual dues of not less than $50.00. They shall be entitled to all of the privileges of Active members and shall have Family privileges as defined in Section 7 of this ARTICLE free of additional charge.

*Section 6.* Honorary members shall be entitled to all of the privileges of sustaining members, except the right to vote and to hold office. They shall be nominated by the Board of Managers and shall be elected by four-fifths of the votes of the members present at any stated meeting of the Institute at which their nomination may be acted upon.

*Section 7.* Family privileges consist of the right to receive a card for each individual in the family of the member and resident with the member entitling the holder to unlimited free admission to the Museum and the Planetarium.

*Section 8.* Members belonging to classes of membership existing prior to the amendment of this ARTICLE shall be reclassified in a manner consistent with their former dues and privileges.

## ARTICLE III

### PAYMENT OF DUES

*Section 1.* The annual fees for membership shall be due and payable on the first day of each month, whichever is nearest to the date of election or as determined by the Board of Managers.

*Section 2.* Any member whose dues are more than two months in arrears shall have all the privileges of membership suspended until such time as all arrears are paid. Should the dues not be paid when they become six months in arrears the said member shall forfeit his membership.

*Section 3.* The Board may remit temporarily in whole or in part the dues of any member either by action in a particular case or by establishing regulations governing certain cases.

*Section 4.* Every person admitted to membership in the Institute shall be considered as liable for the payment of dues until he shall have resigned, been dropped or have been relieved therefrom by the Board of Managers.

*Section 5.* Resignations of memberships shall be made to the Board of Managers in writing, but need not be accepted until all dues and arrears up to date of resignation shall have been paid.

*Section 6.* The privileges and title of Associate member may be obtained for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the Student or Associate Membership dues paid by the member during the preceding ten years, but in no case shall the deduction exceed $50.00.

*Section 7.* The privileges and title of Active Member shall be enjoyed for life or may be obtained for life by a member who has heretofore or who shall hereafter pay therefor, in one year, the sum of $300, except that a person residing permanently at a distance of twenty-five miles or more from Philadelphia may become an Active Member for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the dues paid by the member during the preceding ten years, but in no case

shall the deduction exceed one-half of said payment. Associate Members for life may become Active Members for life by paying therefor in one year the sum of $200.

*Section 8.* Firms, corporations, associations or individuals may nominate and subscribe for the membership dues of groups of members of any class or classes, at the annual dues provided for, subject to the approval of the Board as to any particular nominee. If the dues of these nominees amount to $100 or more in the aggregate, the firm, corporation, association or individual shall be known as an Affiliate of the Institute.

## ARTICLE IV

### MANAGEMENT

*Section 1.* The Institute shall be governed by a board of twenty-four (24) Managers elected by the members.

*Section 2.* The officers, who shall be elected by the Board of Managers, shall be a President, an Executive Vice President, not more than five Vice Presidents, a Secretary, an Assistant Secretary, a Treasurer, and an Assistant Treasurer. The Board of Managers may elect such other officers as it deems necessary.

*Section 3.* At the annual meeting of the Institute, eight Managers shall be elected each year to serve for three years, provided that the Managers now elected, or who may hereafter be elected, shall continue to serve until their successors be appointed.

*Section 4.* All elections of the Institute shall be by letter ballot and no vote may be cast by proxy.

*Section 5.* Nominations for Managers shall be made in writing at the stated meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be posted at the Institute and incorporated (with directions for voting) in a ballot to be sent to each member by the Secretary at least one week before the date of election. Each ballot shall be accompanied by a return envelope addressed "To the Tellers of Election," and provided with a space for the signature of the member voting.

*Section 6.* On the date of the annual meeting, and at an hour previously designated by their chairman, the tellers shall meet at the Institute, and shall count all legal votes that have been received by mail or placed in the ballot box before eight o'clock P.M.; and when the count is completed they shall report to the annual meeting of the Institute the total number of ballots cast, together with the number of votes received by each candidate. Thereupon the presiding officer shall announce the names of the candidates who received the plurality of votes, and shall declare them elected Managers of the Institute for the ensuing terms.

*Section 7.* At the organization meeting of the Board of Managers, the Board of Managers shall elect the officers provided for in Section 2 of ARTICLE IV to serve for one year; and may at said organization meeting or from time to time thereafter elect such other officers as it may determine upon, and shall determine and fix the compensation, if any, to be paid to the officers so elected or appointed by them. The officers who are in office immediately prior to the annual meeting shall continue in office until their successors are elected or appointed by the Board of Managers as herein provided.

*Section 8.* Vacancies occurring in any office may be filled by the Board of Managers by election or appointment of persons to serve until the next annual election.

## ARTICLE V

### BOARD OF MANAGERS

*Section 1.* The Board of Managers shall have general charge and control of the Institute and of the Benjamin Franklin Memorial and The Franklin Institute Museum, and shall consist of twenty-four members elected as provided in ARTICLE IV. The President, the Executive Vice President, the Vice Presidents, the Secretary, the Treasurer, the Chairman of the Committee on Science and the Arts, and the Chairman of the Library Committee shall be *ex officio* members. A quorum of the Board of Managers shall be nine of the elected and *ex officio* members. The Board of Managers may adopt such by-

laws, rules and regulations for the governance of their affairs as are not inconsistent with the Charter and these By-Laws.

*Section 2.* They shall present, through the President, at the annual meeting of the Institute, a report of the condition of the affairs of the Institute.

*Section 3.* They shall hold stated meetings in each month except in July and August.

*Section 4.* Special meetings may be called by the President at his discretion and shall be called by him on written request of the Executive Committee or of any seven members of the Board. In case of his absence or refusal to act, such special meeting shall be called by the Secretary.

*Section 5.* Members who have not attended five regular meetings in the twelve months prior to the stated meeting of the Institute in December, shall be reported thereat as having resigned, unless it be unanimously voted by the Board at that meeting that such member has been absent for sufficient reason.

*Section 6.* All vacancies on the Board of Managers shall be filled by the Board until the next annual meeting of the Institute.

*Section 7.* The Board of Managers may elect an Executive Committee consisting of five of its Members with such authority to act on behalf of the Board as the Board may delegate.

## ARTICLE VI

### DUTIES OF OFFICERS

*Section 1.* The President shall be the executive head of the Institute and, under the supervision of the Board of Managers, shall have general charge of the affairs of the Institute. He shall preside at all meetings of the Institute and of the Board of Managers and shall be, *ex officio*, a member of all standing committees of the Institute.

*Section 2.* The Executive Vice President, under the supervision of the President, shall have immediate charge of all affairs of the Institute other than those matters specifically delegated by action of the Board of Managers pursuant to these By-Laws to the Executive Committee and the Finance Committee. He shall prepare the annual budget, and submit it to the Board of Managers for approval. He shall be an *ex officio* member of all standing Committees of the Institute. He shall report to the President. In case of the disability of the Executive Vice President, the President shall designate another officer or other officers to assume his duties.

*Section 3.* In the absence of the President, the Executive Vice President shall exercise his duties. In the absence of both the President and the Executive Vice President, the Vice Presidents shall exercise the duties of President in order of their seniority in office.

*Section 4.* The Secretary shall be responsible for keeping the minutes of all meetings of the Institute and of the Board of Managers, shall keep the records of the Institute, and shall perform all the duties usually pertaining to the office of Secretary. He shall report to the Executive Vice President. In the absence or disability of the Secretary, the Assistant Secretary shall perform his duties.

*Section 5.* The Treasurer shall have custody of all monies received from the committee on Finance together with all monies received by the Institute from dues, admissions, and other operations of the Institute, depositing them, in the name of the Institute, in such bank or banks as the Board of Managers shall direct and disbursing them by checks signed as the Board of Managers shall likewise direct. He shall keep accounts of the receipts and disbursements and shall report thereon to the Board of Managers as directed. He shall have general supervision of the accounts of the Institute and shall render such financial statements as directed by the Board of Managers. He shall give bond to an amount and with such surety as the Board of Managers shall determine. The Treasurer shall report to the Executive Vice President. In the absence of the Treasurer, the Assistant Treasurers shall, in the order of their seniority in office, perform his duties.

## ARTICLE VII

### BOARD OF COUNCILLORS

The Board of Councillors shall consist of not more than fifty members and shall embrace, as *ex officio* members, the Governor of Pennsylvania, the Chief Justice of the Supreme Court of Pennsylvania, the Mayor of the City of Philadelphia, the President of the City Council of Philadelphia, the President

of the Board of Public Education and the Superintendent of Schools of the City of Philadelphia, the Provost and the President of the University of Pennsylvania, the President of Commissioners of Fairmount Park, the President of the American Philosophical Society, the President of the Poor Richard Club, and the President and Secretary of The Franklin Institute, which two latter shall respectively be Chairman and Secretary of the Board of Councillors. Additional members shall be nominated by the President or Secretary of the Institute and elected by the Board of Managers, at such intervals as the Board of Managers may see fit. The Board of Councillors shall meet at the call of its Chairman or of the Board of Managers for the consideration of any matter that pertains to the welfare of the Institute. The Secretary shall from time to time send to the members of the Board of Councillors a report of the activities and accomplishments of the Institute.

## ARTICLE VIII

### COMMITTEES OF THE INSTITUTE

*Section 1.* There shall be the following Standing Committees of the Institute:

1. Bartol Research Foundation Committee.
2. Biochemical Research Foundation Committee.
3. Committee on Science and the Arts.
4. Endowment Committee.
5. Finance Committee.
6. Library Committee.
7. Meetings Committee.
8. Membership Committee.
9. Museum and Memorial Committee.
10. Publications Committee.
11. Committee on Research.

*Section 2.* Each standing committee of the Institute shall consist of the number of members determined on by the Board of Managers and composed of members of the Board of Managers and/or non-members as may be desirable, except that the Committee on Science and the Arts shall consist of not less than sixty nor more than seventy-five members, and except that the Bartol Research Foundation shall consist of not less than ten nor more than fifteen members as provided for in Section 7 of this ARTICLE.

*Section 3.* The members of each committee shall serve one year, except that members of the Committee on Science and the Arts shall serve three years.

*Section 4.* The members of each committee, other than *ex officio* members specifically provided for by these By-Laws, shall be appointed by the President and approved by the Board of Managers at the organization meeting of the Board following the annual meeting except that each year but one-third of the total number of members of the Committee on Science and the Arts shall be so appointed and approved.

*Section 5.* The Chairman of each Committee shall be designated by the President, except that the Library Committee and the Committee on Science and the Arts shall select their chairmen. A chairman who is not an elected or *ex officio* member of the Board of Managers shall have the privilege of the floor at meetings of the Board but shall not have the right to vote.

*Section 6.* Each Committee shall report monthly to the Board of Managers through its Chairman. A copy of their report shall be in the hands of the Executive Vice President one week before the monthly meeting of the Board.

*Section 7.* The Bartol Research Foundation Committee shall consist of not less than ten nor more than fifteen members, including the President and the Executive Vice President of the Institute and the Director of the Bartol Research Foundation as *ex officio* members and nine members of the Institute of whom not more than four shall be members of the Board of Managers. The Committee shall, after consultation with the Director of the Bartol Research Foundation and subject to the approval of the Board of Managers, collaborate with the Executive Vice President in carrying out the purposes and determining the policies of the Bartol Research Foundation. It shall appoint the Director and the Staff of the Laboratories, subject to the approval of the Board of Managers. The Director of the Bartol Research Foundation, to whom all other employees of the Foundation shall report, shall have charge of the scientific activities of the Foundation and shall report on all matters to the Committee or to the Executive Vice President, as the Committee may elect.

*Section 8.* The Biochemical Research Foundation Committee shall report to the Board of Managers from time to time on the condition of the affairs and operation of said Foundation and shall be available to the Director of the Biochemical Research Foundation and its Advisory Council for collaboration and advice.

*Section 9.* The Committee on Science and the Arts shall investigate current discoveries, inventions and other achievements in the sciences and their application in the mechanical and industrial arts with a view of affording such recognition as may lie within the power of the Institute to bestow. The Committee shall exercise independent judgment therein. In all matters of a general or administrative nature the Committee shall collaborate with the Executive Vice President.

*Section 10.* The Endowment Committee shall, in collaboration with the Executive Vice President, encourage the gift of funds to the Institute.

*Section 11.* The Finance Committee shall have the custody and control of all the securities and investments of the Institute with full power and authority to buy and to sell, and to invest and reinvest the same; including the power to satisfy mortgages and extinguish ground rents, and to direct the placing of all such insurances on Real Estate held for investment as it may deem necessary, and to make such improvements, repairs and alterations to such Real Estate as it may deem necessary. It shall have power to authorize the proper officers of the Institute to execute the necessary papers to effect all purchases, sales and assignments of property other than real estate; to transfer registered securities; to subscribe to bond-holders' agreements to plans of reorganization involving any securities held by the Institute or in which it has an interest; and to do all such acts as are necessary in pursuance of the foregoing powers. It may, with the approval of the Board of Managers, appoint a Trust Company of the City of Philadelphia to act as Fiscal Agent under the direction of the Committee.

The Finance Committee shall keep a record of all its acts and proceedings, which shall be communicated to the Board of Managers.

*Section 12.* The Library Committee, in collaboration with the Executive Vice President, shall be of counsel and advice in the purchase of books and publications suitable to and consonant with the purposes of the Library; and in other matters pertaining to the Library.

*Section 13.* The Meetings Committee shall, in collaboration with the Executive Vice President, secure for presentation before the Institute papers dealing authoritatively with subjects of import in the fields of science, engineering and industry.

*Section 14.* The Membership Committee shall, in collaboration with the Executive Vice President, promote the several memberships defined in ARTICLE II of these By-Laws.

*Section 15.* The Museum and Memorial Committee shall, in collaboration with the Executive Vice President, be of counsel and advice to the Director of the Benjamin Franklin Memorial and The Franklin Institute Museum, and with the scientific staff of the Museum concerning the activities and the exhibits of both the Museum and the Memorial.

*Section 16.* The Publications Committee, in collaboration with the Executive Vice President, shall be of counsel and advice with regard to all publications of the Institute other than publications of the Bartol Research Foundation.

*Section 17.* The Committee on Research shall, in collaboration with the Executive Vice President, be of counsel and advice to all research activities of the Institute except those of the Bartol and Biochemical Foundations.

*Section 18.* Each committee shall organize and adopt rules as it sees fit, subject to the provisions of these By-Laws and the approval of the Board of Managers.

## ARTICLE IX

### MEETING

*Section 1.* The Institute shall hold stated meetings on the third Wednesday of each month, except in June, July, August and September, at 8:15 P.M.

*Section 2.* Special meetings shall be called by the President upon the order of the Board

of Managers or within ten days upon the written application of twenty-four voting members of the Institute.

*Section 3.* Twenty-four members shall constitute a quorum at any stated or special meeting.

*Section 4.* The Annual Meeting of the Institute shall be held on the third Wednesday of January of each year at 8:15 P.M. Elections at Annual Meetings shall not result from the poll of less than forty-eight ballots cast in accordance with ARTICLE VI, Section 6, of these By-Laws.

## ARTICLE X

### AMENDMENTS

These By-Laws may be altered or amended at any stated meeting of the members of the Institute, provided notice in writing, signed by two members, of the proposed alteration or amendment, shall be given to the Board of Managers two months prior to the said meeting, except that amendments to ARTICLE I, relating to capital stock, must be ratified subsequently by a majority of the stock represented at a meeting specially called for this purpose.